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Numerische Lineare Algebra 2 – Scaling & Squaring Methods.

Algorithm 1 Scaling & Squaring Method for $\exp(A)$

Input: $A \in \mathbb{C}^{n \times n}$
Output: $F = \exp(A)$.
1: Find $s \in \mathbb{N}$ such that $\|A/2^s\|_1 \leq \theta_{13} \approx 5.4$
2: Scaling: $A \leftarrow A/2^s$
3: Evaluate [13/13]-Padé approximation $X = r_{13}(A)$.
4: Squaring: $F \leftarrow X^{2^s}$.

Algorithm 2 Inverse Scaling & Squaring Method for $\log(A)$

Input: $A \in \mathbb{C}^{n \times n}$
Output: $F = \log(A)$.
1: Compute Schurform $Q^*AQ = R$.
2: Find $s \in \mathbb{N}$ such that $\|R^{\frac{1}{2^s}}\| \approx I$
3: Successively compute square roots by adaptation of Schur-Parlett: $R \leftarrow R^{\frac{1}{2^s}}$
4: Evaluate Padé approximation $X = r_m(R - I)$.
5: Scaling & back transformation: $F \leftarrow 2^s QXQ^*$.

Remarks:

- In practice, s, m are chosen adaptively based on $\|A\|$ (or $\|R - I\|$ for inverse S&S). The value $m = 13$ has been proven a good candidate for S&S in double precision arithmetic.
- Schurform in inverse S&S can be avoided, but then the square roots have to be computed differently, e.g., by Newton's method and variants thereof (Section III.5.6).

References

[1] Higham, N.: *The Scaling and Squaring Method for the Matrix Exponential Revisited* SIAM J. Matrix Anal. & Appl., 26(4), 1179-1193, 2006.

[2] Higham, N.: *Functions of Matrices: Theory and Computation (Chapters 10&11).* Society for Industrial and Applied Mathematics, Philadelphia, PA, USA (2008)

[3] Al-Mohy, A. and Higham, N.: *A New Scaling and Squaring Algorithm for the Matrix Exponential* SIAM J. Matrix Anal. & Appl., 31(3), 970-989, 2009.