Otto-von-Guericke-Universität Magdeburg
Max-Planck-Institut für Dynamik komplexer technischer Systeme
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Numerical Linear Algebra – Question sheet 2.

Please hand in your completed problems on 22.11.2019.

Problem 1 **(7 points)**

Consider the $n \times n$ matrix of the following form

$$A = \begin{bmatrix} d & s & & \\ s & d & \ddots & \\ & \ddots & \ddots & s \\ & & s & d \end{bmatrix}, \quad d, s \in \mathbb{R}.$$

(a) Verify that $\lambda_j = d + 2s\cos(j\theta)$ with $\theta = \frac{\pi}{n+1}$ and

$$q_j = [\sin(j\theta), \sin(2j\theta), \ldots, \sin(nj\theta)]^T.$$

are the eigenvalues and -vectors of A.

(b) Show that if $d = 2, s = -1$ (discretized 1D Poisson problem), the eigenvalues can be expressed as $\lambda_j = 4\sin^2(\frac{j\theta}{2})$.

(c) Now consider the weighted Jacobi iteration and show that the iteration matrix can be written as $G_\omega = I - \omega D^{-1}A$. For which ω does the weighted Jacobi method converge when applied to the above A with $d = 2, s = -1$?

Problem 2 **(4 points)**

Prove that for the Chebyshev polynomials $\tau_0(t) = 1$, $\tau_1(t) = t$, $\tau_{k+1}(t) = 2t\tau_k(t) - \tau_{k-1}(t)$ the following identity holds

$$\tau_k(t) = \frac{1}{2}\left[\left(t + \sqrt{t^2 - 1}\right)^k + \left(t - \sqrt{t^2 - 1}\right)^k\right].$$

Problem 3 **(5 points)**

We consider the Peaceman-Rachford ADI iteration described by

$$(H + \rho I)u_{k+1/2} = (\rho I - V)u_k + b \tag{1}$$
$$(V + \rho I)u_{k+1} = (\rho I - H)u_{k+1/2} + b, \tag{2}$$

which solves the linear system $(H + V)u = b$.

(a) Show that this can be written as $u_{k+1} = Gu_k + f$. What is G and f?

(b) Verify the splitting $A = M - N$ with $M = \frac{1}{2\rho}(H + \rho I)(V + \rho I)$ and $N = \frac{1}{2\rho}(H - \rho I)(V - \rho I)$. Show also that f from part (a) can be written as $M^{-1}b$.

Problem 4 **(4 points)**

Compute QR factorization of the following matrix by using Househloder transformations

$$A = \begin{bmatrix} 50 & 188 & -160 \\ -50 & 244 & 220 \\ -50 & -204 & -220 \\ -50 & -252 & -760 \end{bmatrix}.$$