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Computational Methods for Systems and Control Theory

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Numerical Linear Algebra – 1. Übung

Exercise 1 (6 important Matrix factorizations)

a) The following six matrix factorizations are extremely important in numerical linear algebra:

- **i)** Cholesky-,
- **ii)** LU with partial pivoting,
- **iii)** QR,
- **iv)** Spectral,
- **v)** Schur and
- **vi)** Singular value decomposition (SVD).

Mention the properties of these factorizations and discuss their existence and uniqueness. Which algorithms do you know to compute each individual decomposition

Exercise 2 (Matrix manipulation)

The following operations are performed on a 4×4 matrix A:

- **a)** Multiplying the 2nd column by 2
- **b)** Multiplying the 1st line by $\frac{1}{2}$
- **c)** Adding line 2 to the line 4
- **d)** Swap columns 1 and 2
- **e)** Subtract line 1 from all other lines
- **f)** Exchange lines 3 and 4
- **g)** Remove column 2 (the column dimension is reduced by 1)

- Write the previous operations in matrix form.
- Write the matrix resulting from the previous operations as a product of 3 matrices UAV

Exercise 3 (QR decomposition)

Show that the economic QR decomposition $A = QR$ is unique if $A \in \mathbb{R}^{m \times n}$ $(m \geq n)$ has full column rank. Q has orthonormal columns and R is upper triangular matrix. In addition, show that R corresponds to the upper triangular factor G of the Cholesky factorization of $A^\top A$
Compute the economic QR decomposition of the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 2 \\ 1 & 5 \end{bmatrix}$$

Exercise 4 (Givens rotations)

We say that two matrices $A, B \in \mathbb{R}^{n\times n}$ are similar if there exists an invertible matrix $S \in \mathbb{R}^{n\times n}$ such that $A = SBS^{-1}$. We call S in that case a similarity transformation matrix. These matrices play an important role in numerical linear algebra. They transform A into equivalent form that has a desirable properties. Givens rotation matrix is defined as:

$$G(j,k,\theta) = \begin{bmatrix} 1 & \dots & 0 & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & c & \dots & s & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & -s & \dots & c & \dots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & 0 & \dots & 1 \end{bmatrix},$$

where, $c = \cos(\theta)$ and $s = \sin(\theta)$. These matrices are used to eliminate entries in vectors and matrices. Derive the representations for c and s, so that the matrix G can be used to eliminate the entry at position k of a vector x with the help of the entry at position j. Show that the matrix G is an orthogonal matrix.