# Reduced Order Models for Aerodynamic Applications, Loads and MDO

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**3rd Workshop on Model Reduction of Complex Dynamical Systems**
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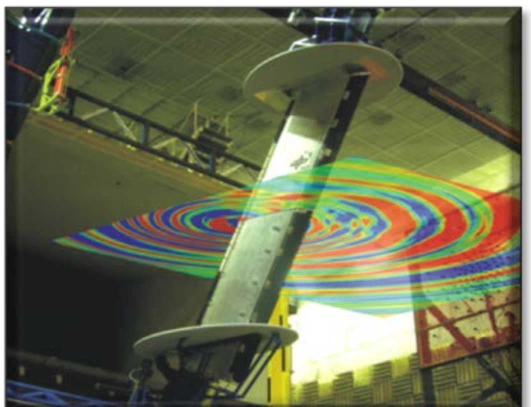
Odense, Denmark


DLR

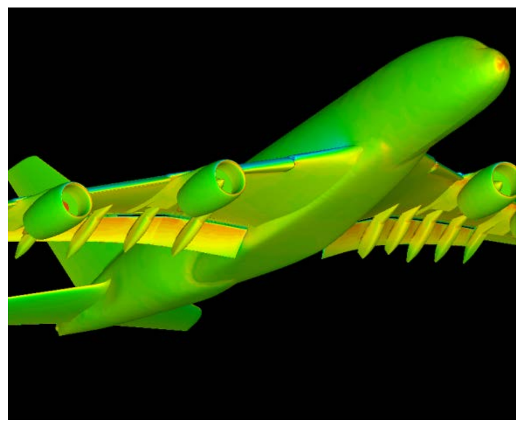
# German Aerospace Center (DLR)

**DLR is the national aeronautics and space research center of Germany, Germany's space agency and the largest project management agency**

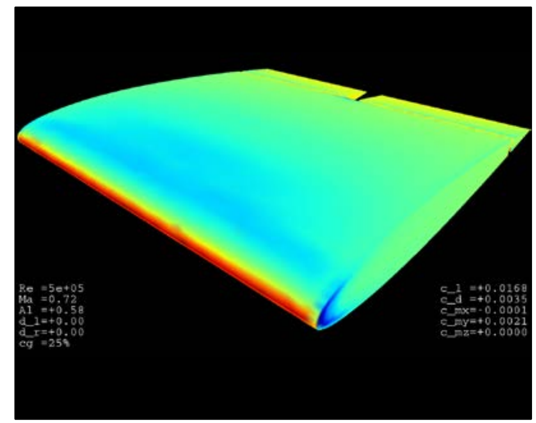
**Institute of Aerodynamics and Flow Technology** › **C²A²S²E Department** › **Aerodynamic Surrogates and Optimization Group**



Research in aerodynamics & aeroacoustics of airplanes, helicopters, wind turbines and aerothermodynamics of space vehicles



Numerical methods for multidisciplinary simulation and optimization of aircraft




Optimization, MDO, surrogate- and reduced order modeling, UQ and robust design based on high-fidelity CFD

**Software: TAU, FlowSimulator, SMARTy used by acedemia, research & industry**

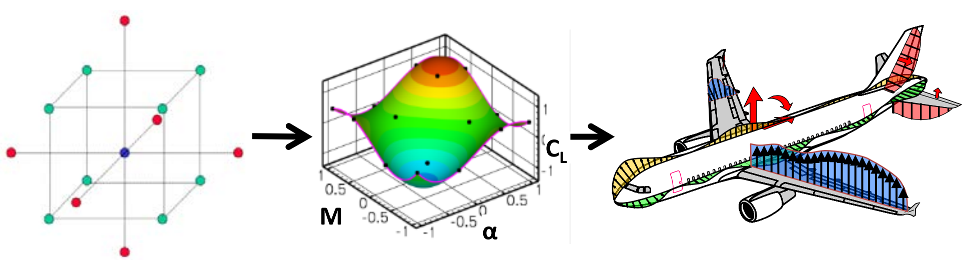
# Software | **Surrogate Modeling for AeRo Data Toolbox**

SMARTy

**SMARTy is a modular Python package for rapidly predicting aerodynamic data based on high-fidelity CFD**

## Key features

- Design of experiments
- Adaptive sampling
- Surrogate Modeling
- Variable-Fidelity Modeling
- **Reduced Order Modeling**
- Dimensionality reduction
- CFD interface via FlowSimulator
- Parallelized




## Applications

- Surrogate-based shape optimization
- Uncertainty quantification
- Robust & inverse design optimization

- Fusing experimental + CFD data
- Optimal sensor placement
- Wind-tunnel corrections and support

- Accelerating CFD computations
- Virtual flight testing
- **Rapid loads prediction across the flight envelope** (and structural sizing)

# Software | **Surrogate Modeling for AeRo Data Toolbox**

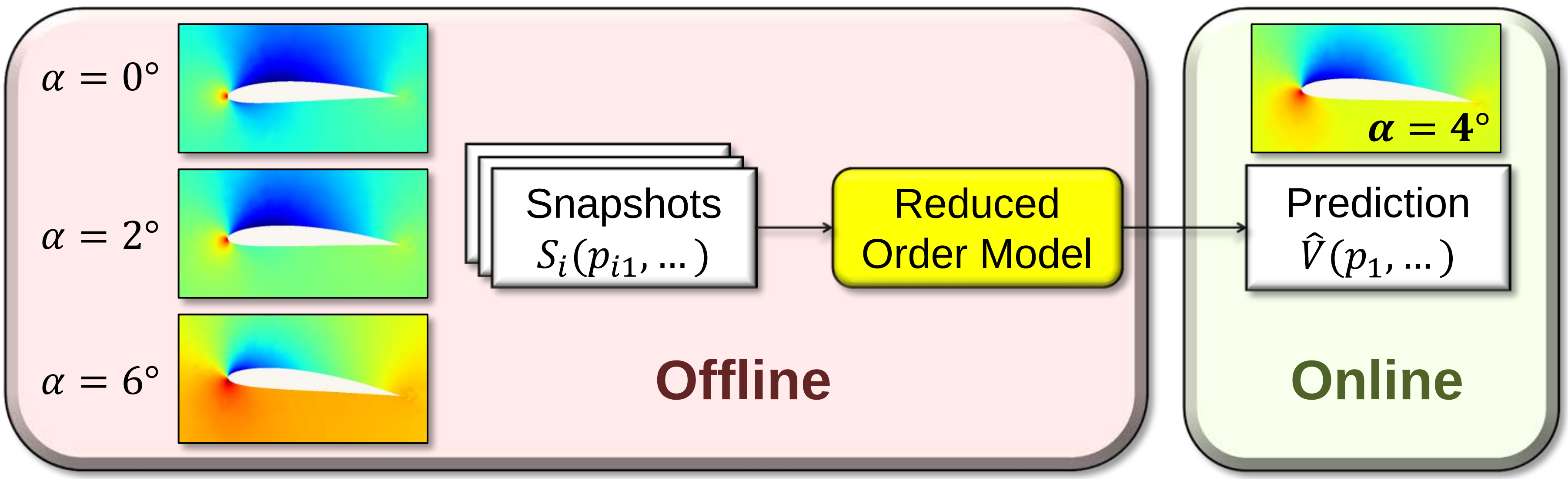
SMARTy

## Reduced Order Models

- Linear and nonlinear methods
- Intrusive and nonintrusive
- Steady and unsteady flows
- Subsonic and transonic speeds

## Users and collaborators

- **Industry:** Airbus, MBDA
- **Research:** DLR, DNW, ARA
- **Academia:** diff. EU universities



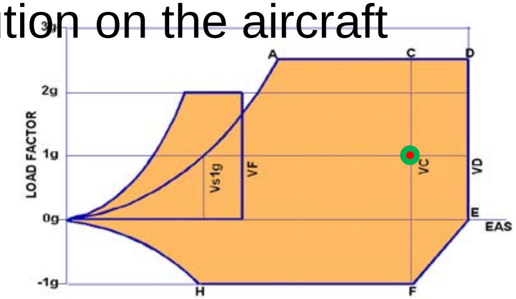

**MODRED 2013:**
- T. Franz: Nonlinear Reduced Order Modeling for Transonic Flows via Manifold Learning
- R. Zimmermann: A parametric ROM for the linear frequency domain approach to time-accurate CFD

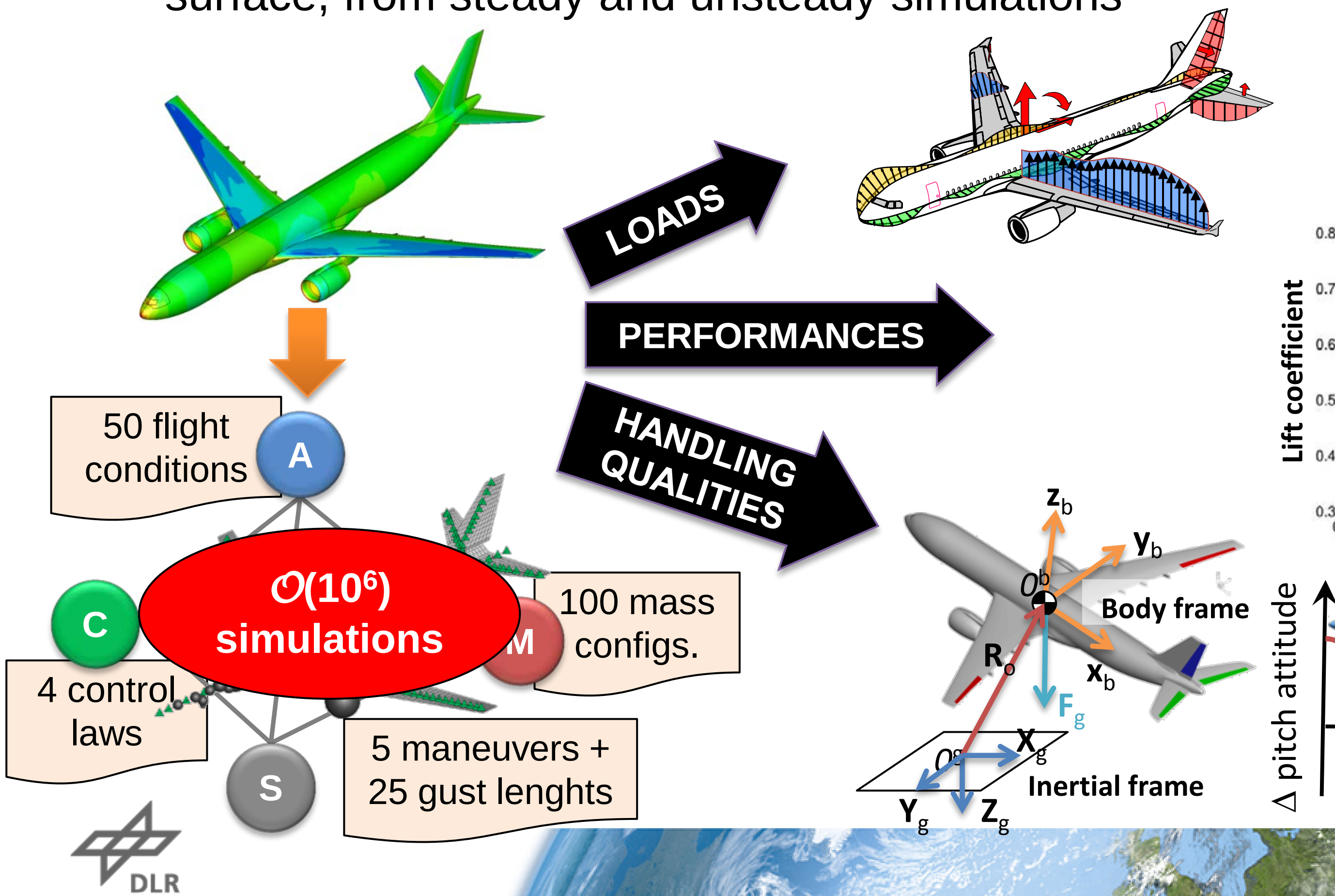
# Outline

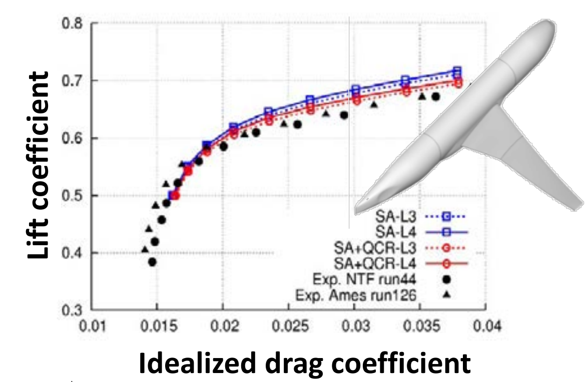
1. Motivation and objectives
2. Intro to our favorite Reduced-Order Modeling (ROM) methods
3. Applications:
   - multi-disciplinary optimization (MDO)
     - static aeroelastic predictions
     - loads prediction and structural sizing
     - efficient selection of critical load cases
   - unsteady aerodynamics and maneuvers
4. Summary & outlook

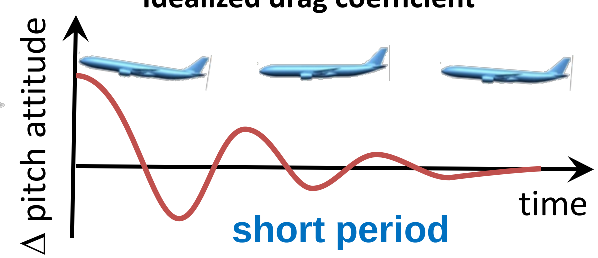
# Motivation

- From design to certification of an aircraft many aerodynamic data are needed – for the entire flight envelope –
- **Aerodynamic data** → pressure and shear stress distribution on the aircraft surface, from steady and unsteady simulations



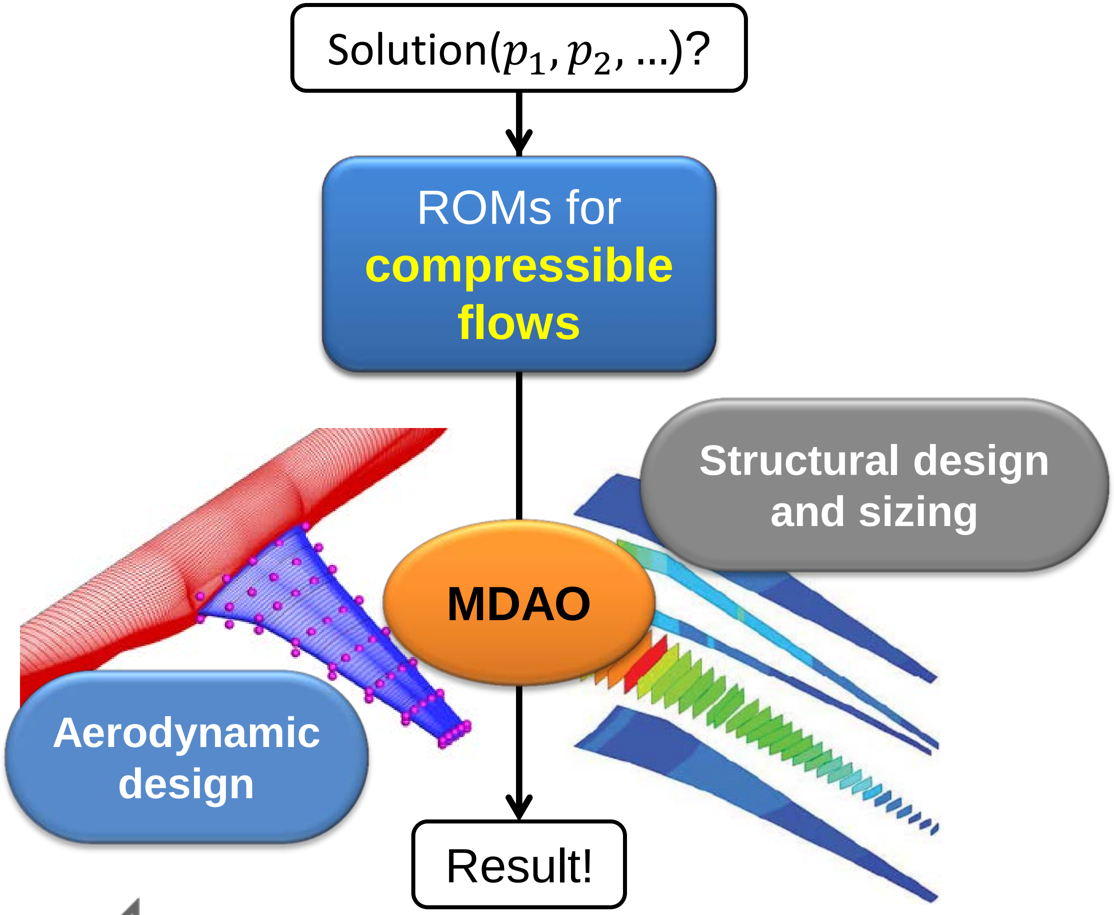



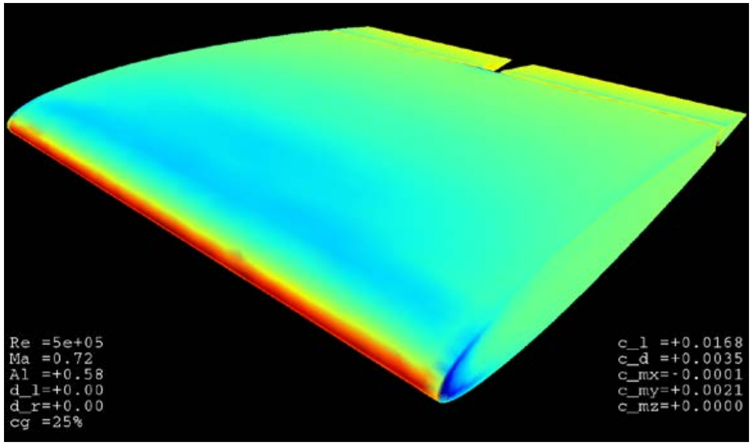



# Objectives

**Goal:** provide predictions of the aerodynamics with lower evaluation time and storage than the original CFD model based on high-fidelity CFD data



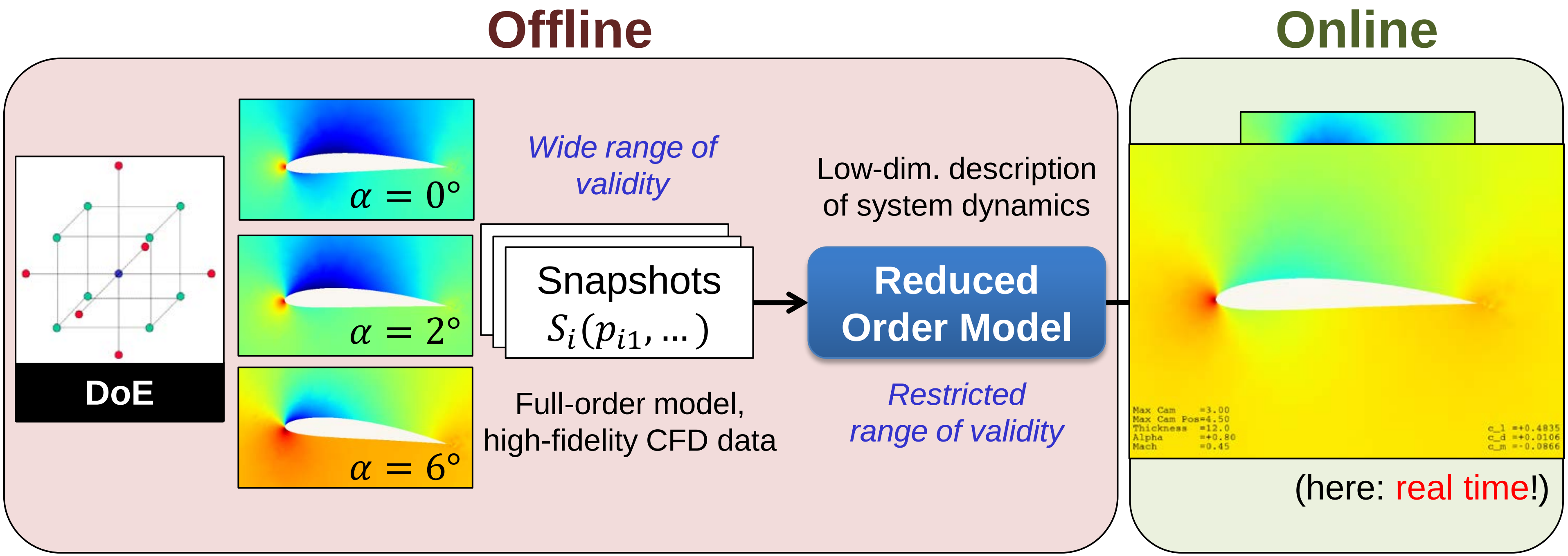

To reduce the computational complexity, **ROMs based on Hi-Fi CFD** should replace (e.g. in the MDO) high quality models and methods




(here: real time!)

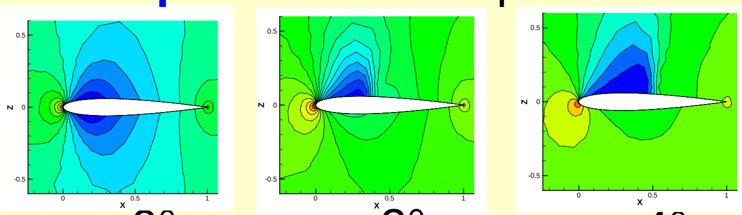
# Reduced-Order Modeling (ROM)

- Reduced-order models operate on parameterically generated data (snapshots)
  - surface quantities: pressure and shear stress $c_p$, $c_f$
  - volume quantities: primitive variables $\rho$, $v_i$, $p$, $T$
- Parameters can be related to the **flow** (e.g. angle of attack $\alpha$, Mach number $Ma$) or to the **geometry**



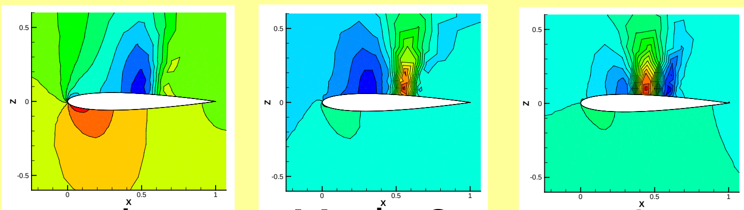
# POD-based Reduced-order Modeling (steady RANS)

**1. Input:** $m$ CFD snaphots $W^m$



α=0° α=2° α=4°

**2. POD Basis:** $m$ Modes $U^m$



Mode 1 Mode 2 Mode 3

RIC>0.9999

**3. Order Reduction**
Select $\tilde{m}$ POD components
with largest information content

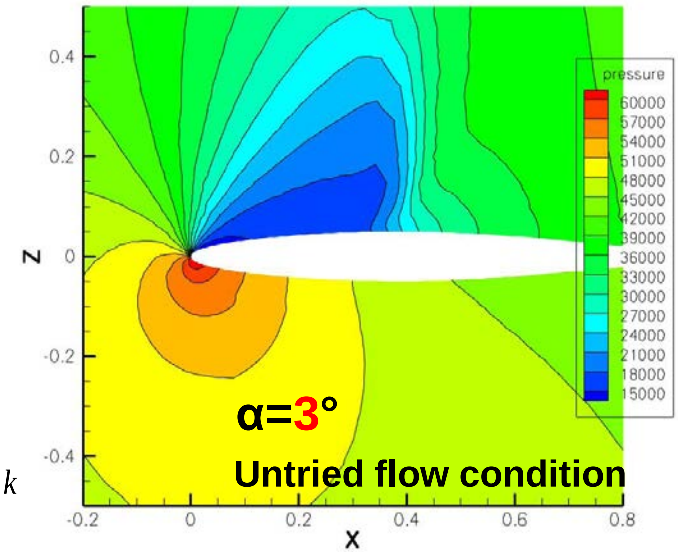
**4. Interpol./Optimization Step**
Determine POD-ROM coefficients $\boldsymbol{a_k}$
such that defect of POD solution $W(a)$
to governing equations is minimized

$$\min_{a=(a_1,\ldots,a_{\tilde{m}})}! \left\| \mathrm{Res}(W(a)) \right\|^2$$

Requires full flow field sol.

$$W = \sum_{k=1}^{\tilde{m}} a_k U^k$$

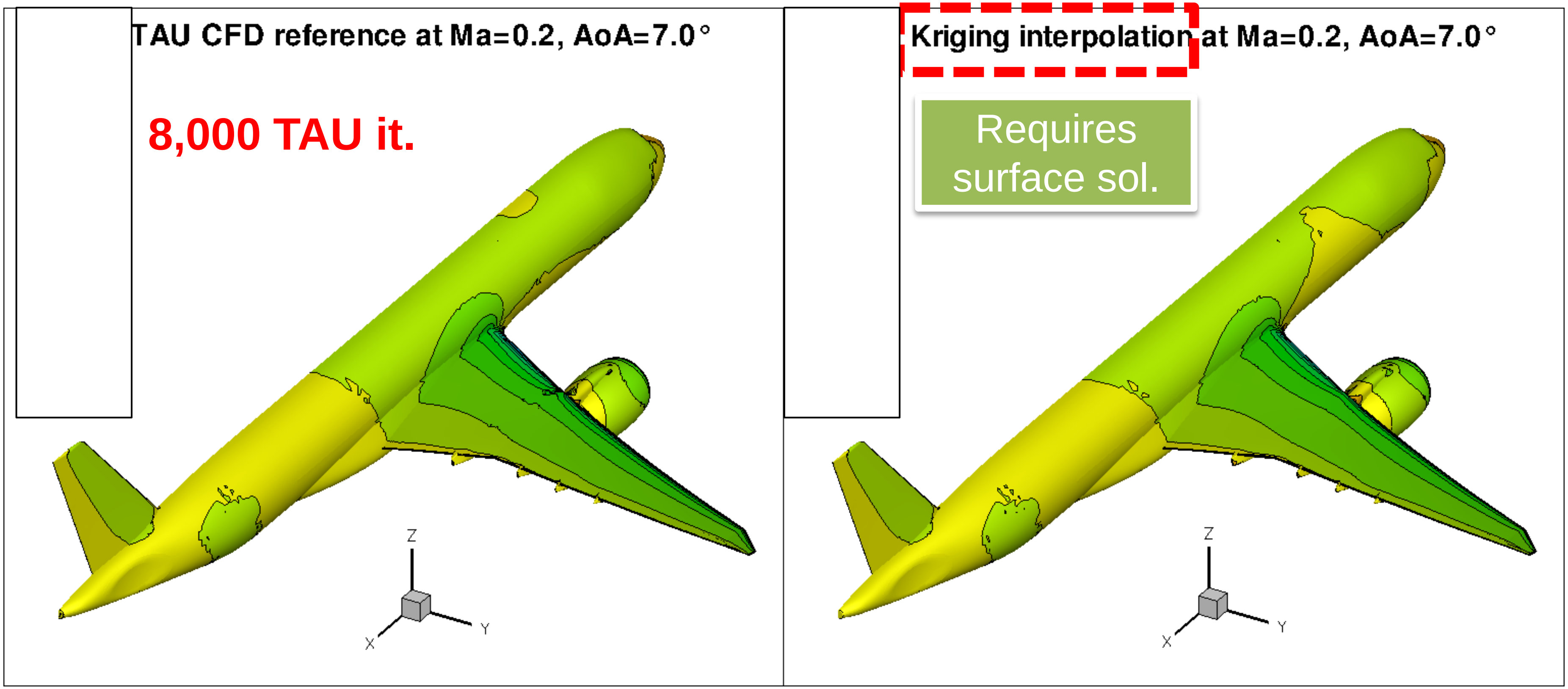
**5. Output:** approximated flow field



# POD-based Reduced-order Modeling (steady RANS)

- Transport a/c config., grid size: 8,898,749
- 4 Snapshots at **α = [-1°,0°,1°,2°]** with TAU code
- Approximation at **α = 7°** (extrapolation)

Navier-Stokes ROM
subsonic




*By Ralf Zimmermann*

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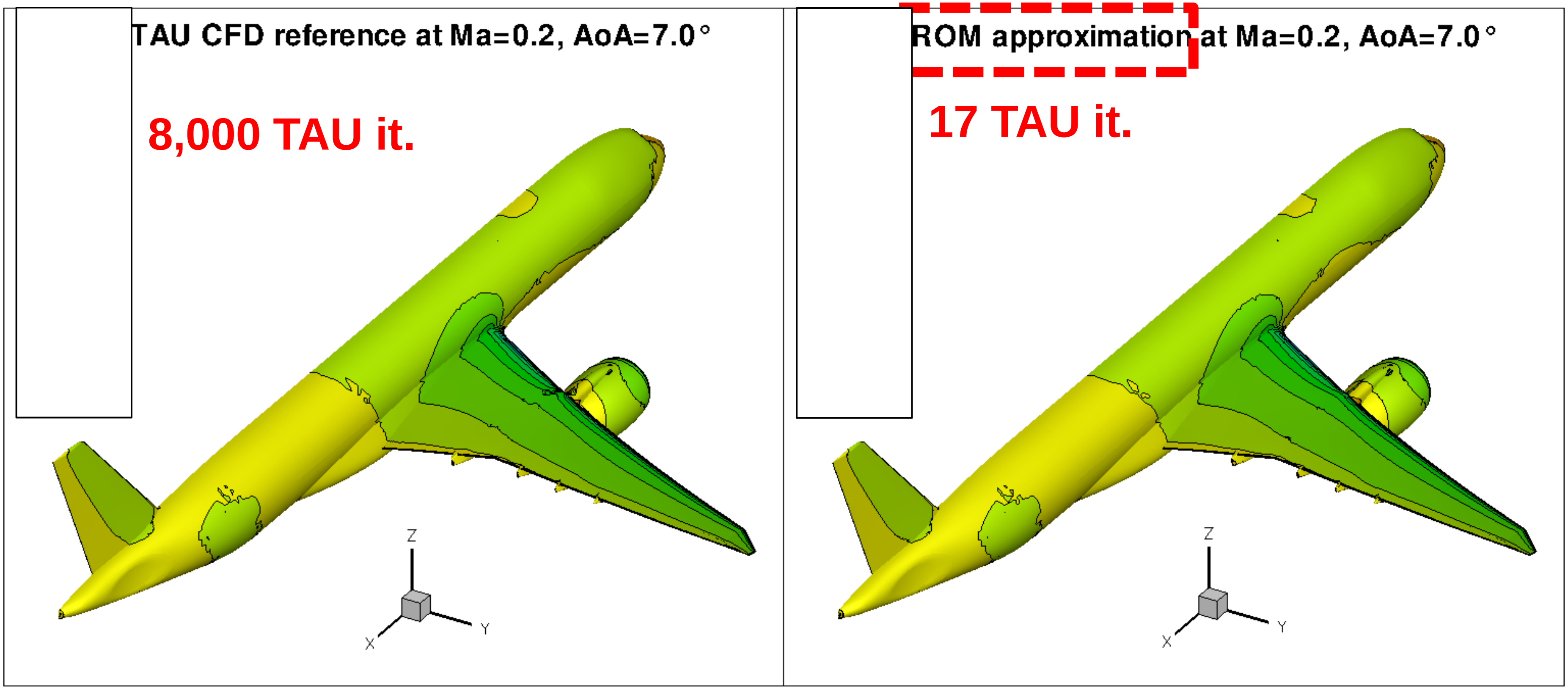
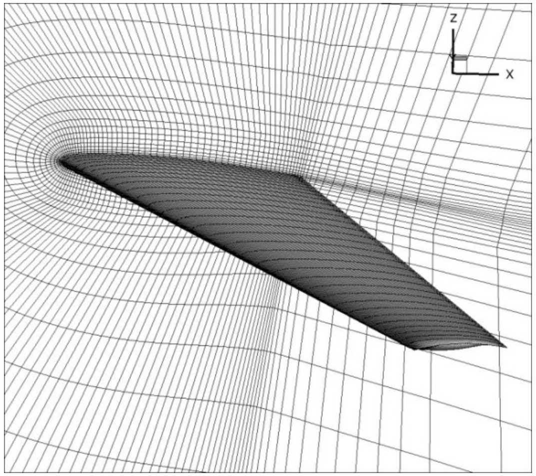



**ROM speed-up by factor: 470**

*By Ralf Zimmermann*

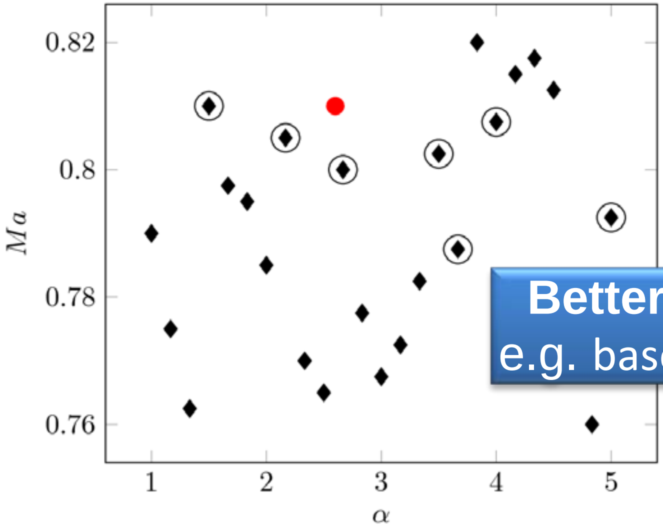
# Reduced-order Modelling via Isomap

**Euler ROM**
**transonic**



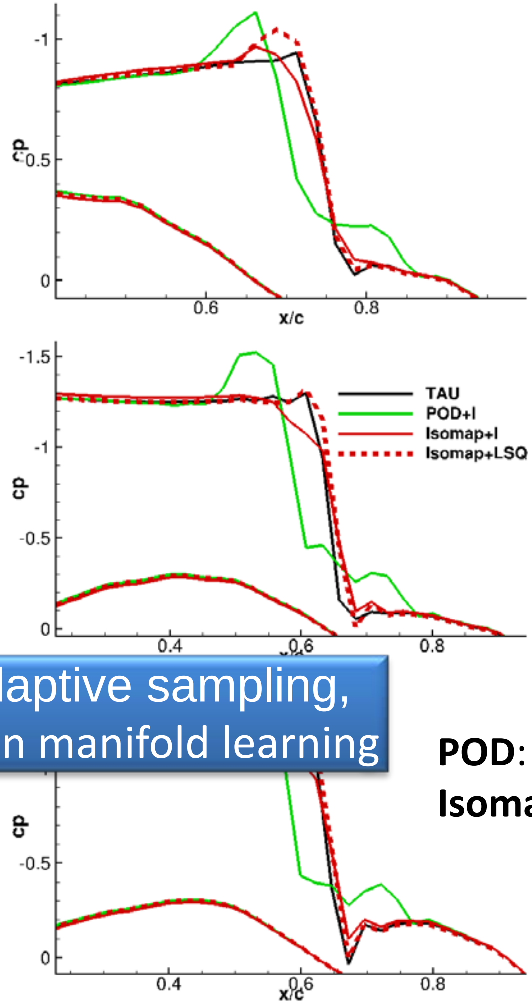

- Grid: 237373 nodes
- CFD Code: TAU Euler

- Latin Hypercube Sampling
- 25 Snapshots

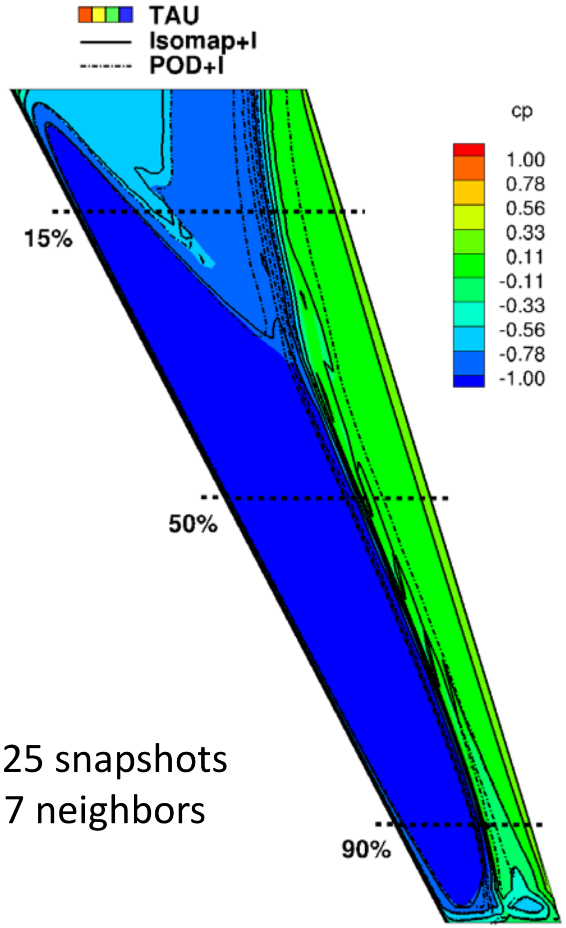



◉ Nearest neighbors on solution manifold

● Prediction point (M=0.81, α=2.6°)




**Better:** adaptive sampling, e.g. based on manifold learning

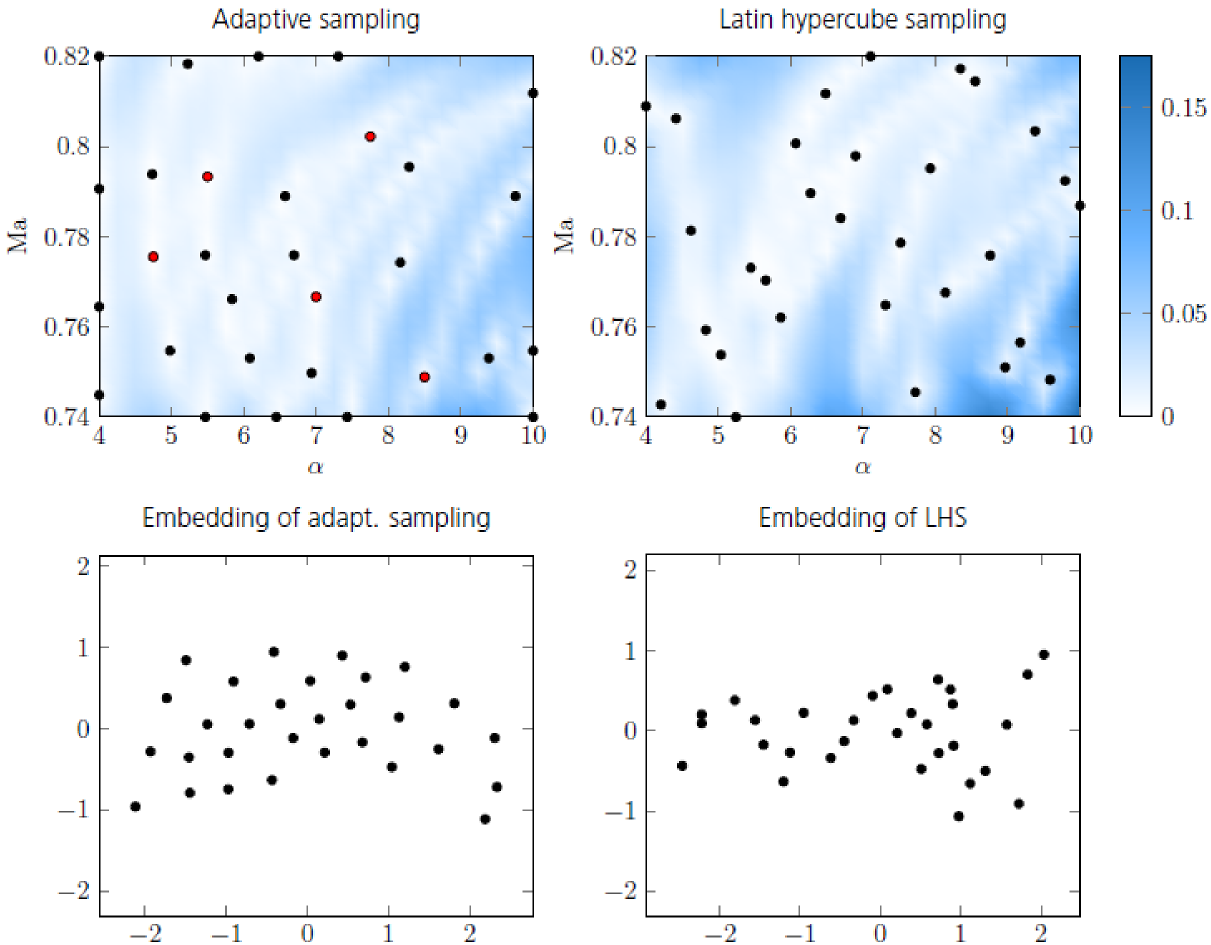



**POD**: all 25 snapshots
**Isomap**: 7 neighbors

$M = 0.81, \alpha = 2.6°$

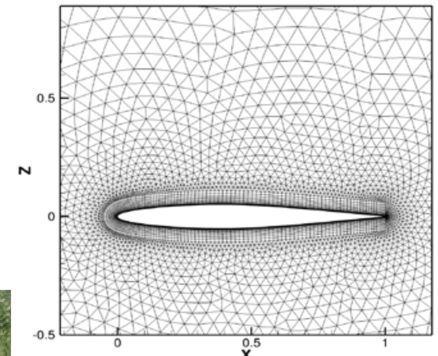
*By Thomas Franz*

# Adaptive sampling (NACA 64A010 test case)




| | AS |
|---|---|
| $e_{max}$ | 0.09296 |
| $e_{mean}$ | 0.02335 |
| $\sigma$ | 0.01662 |

| | LHS |
|---|---|
| $e_{max}$ | 0.18009 |
| $e_{mean}$ | 0.03126 |
| $\sigma$ | 0.02626 |




$\Rightarrow$ adaptive sampling leads to a more space-filling design

R. Zimmermann, T. Franz: On the notion of distance in data-driven manifold learning and surrogate modeling, MORML '16, Stuttgart

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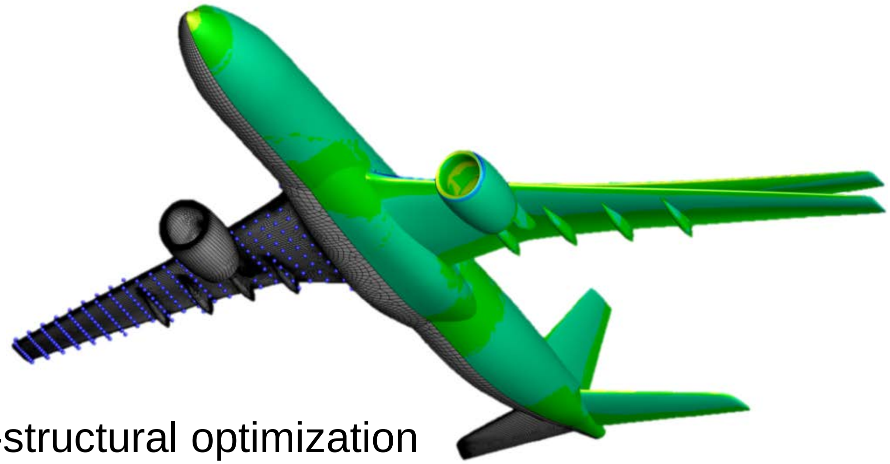
3. **Applications:**
   - multi-disciplinary optimization (MDO)
     - static aeroelastic predictions
     - loads prediction and structural sizing
     - efficient selection of critical load cases
   - unsteady aerodynamics and maneuvers
4. Summary & outlook

# Why use ROMs for multidisciplinary optimization (MDO) based on high-fidelity models?

- Multidisciplinary optimization (MDO) uses optimization methods to solve design problems incorporating a number of disciplines
- Optimum is superior to the design found by optimizing each discipline sequentially, since MDO can exploit the interactions between the disciplines
- Including all disciplines simultaneously significantly increases the complexity of the problem
- Complex multi-disciplinary interactions may occur, cannot be evaluated by low-fidelity models
- High-fidelity CFD / CFD-CSM “mature” but costly in many-query scenarios

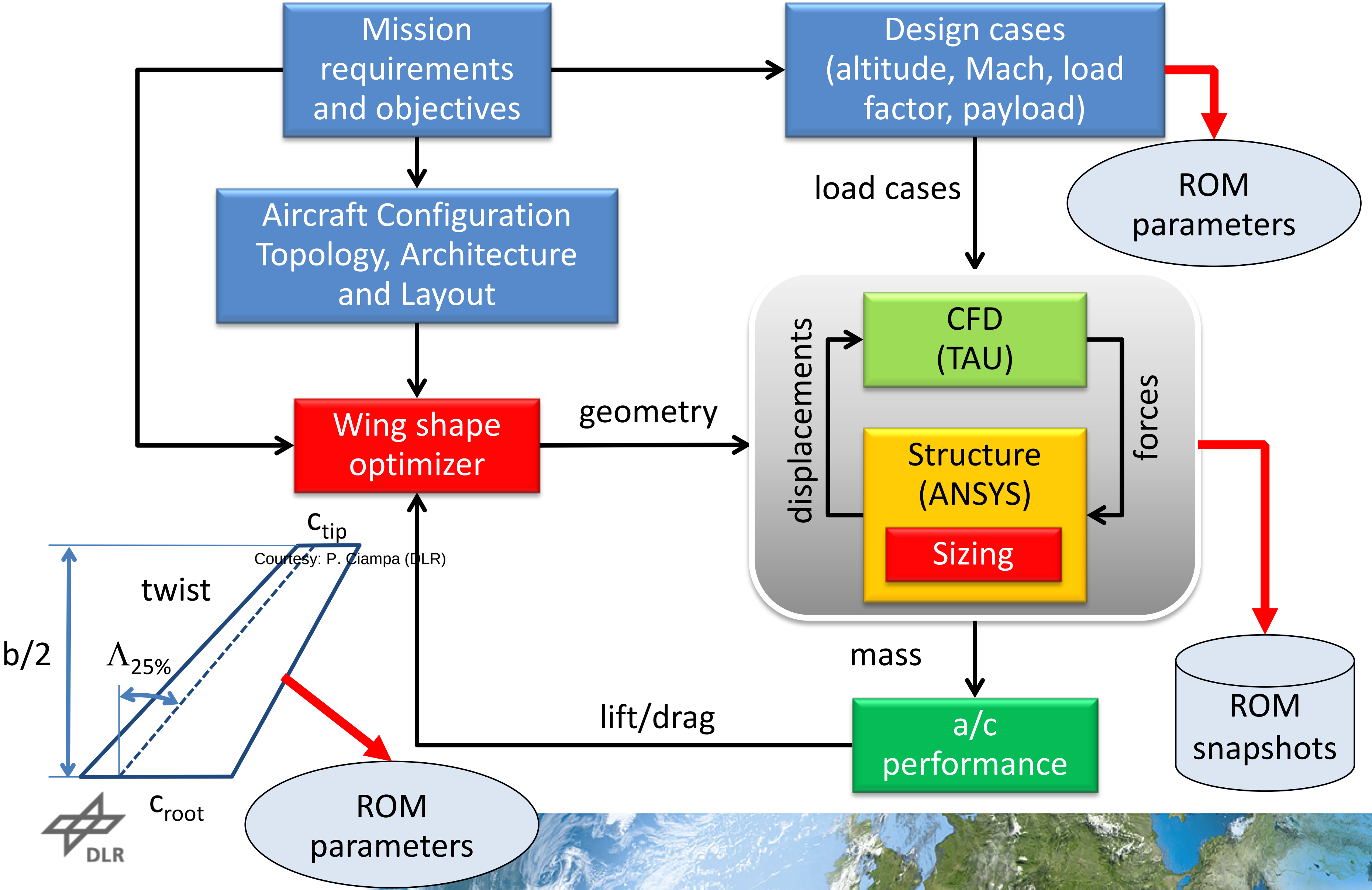
→ **make use of parametric aerodynamic / aeroelastic ROMs based on hi-fi CFD**



Aero-structural optimization

DLR

# Multidisciplinary optimization of transport aircraft



# Load Case Definitions

- **5** Mass configurations
  - 1: OEM (operational empty mass)
  - 2: MZFM (max. zero fuel mass)
  - 3: MFM (max. fuel mass)
  - 4: MTOM (max. take off mass, max. Fuel)
  - 5: MTOM (max. take off mass, max. Payload)

- **6** Design speeds [CS25]
- **6** Flight levels (0 – 13.100 m)

- **7** Stationary trim manoeuvers
  - Symmetric (Pull-up / Push-down)
  - Yawing, Rolling

- **40** Gust encounters [CS25.431(a)]
  - 10 gust gradients
  - 4 directions

**50.400 Load Cases**

(for one a/c configuration!)

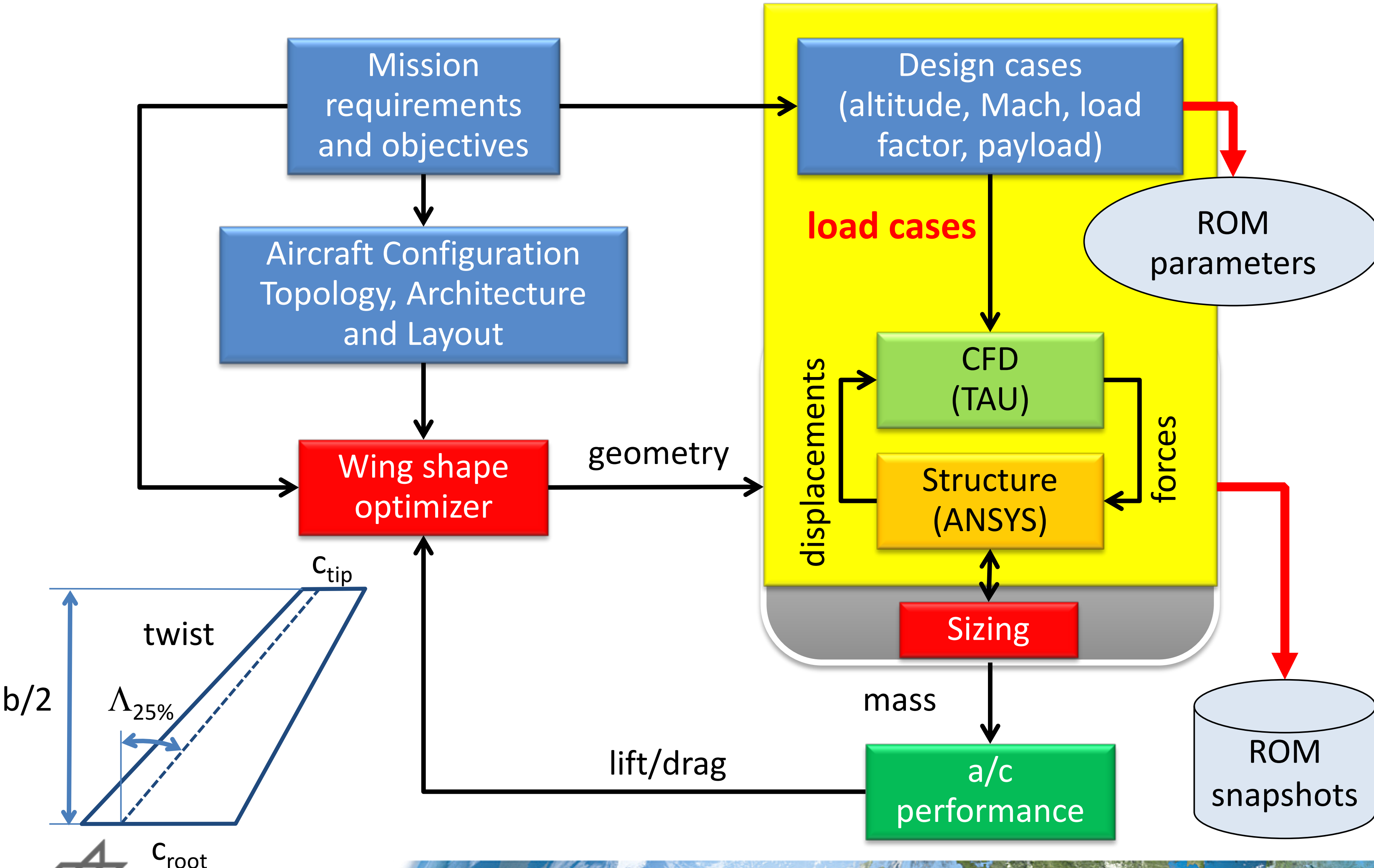
**Full MDO**

$O(10^7)$ CFD* sim.

*: nowadays, fast low-fidelity methods and corrections are used rather than CFD

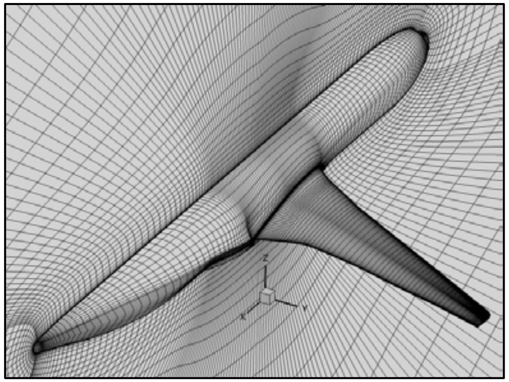
Courtesy T. Kier, DLR- SR

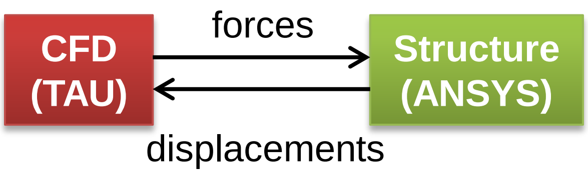
# MDO: static aeroelastic loads predictions using ROMs



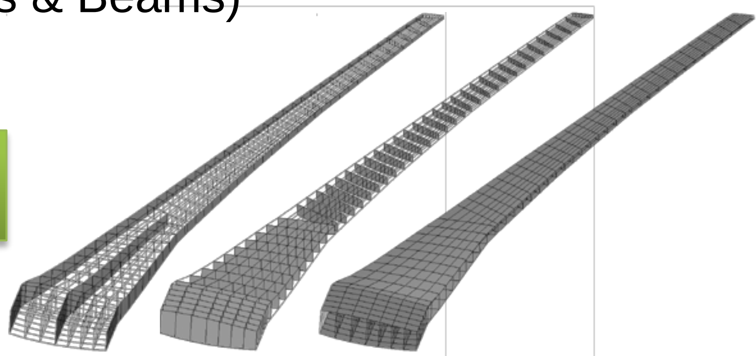
# Static aeroelastic loads prediction of the XRF1 aircraft

CFD Model (RANS, 1M grid nodes)





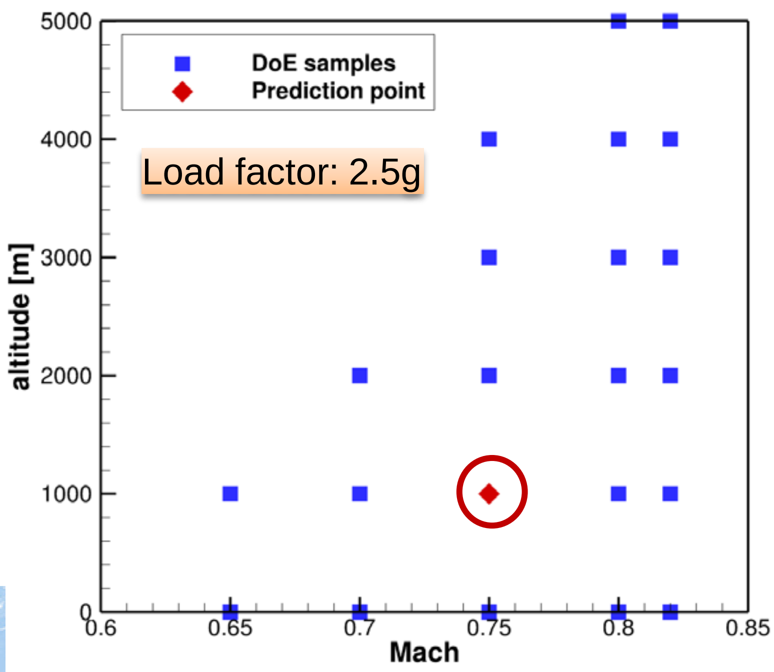

Finite Element Model (2167 Elements) (Shells & Beams)



ROM (POD+I, Isomap+I) from 21 Snapshots for
- a load factor of 2.5g,
- a payload of 55t and 61t fuel mass

**Varying parameters:** Mach, altitude

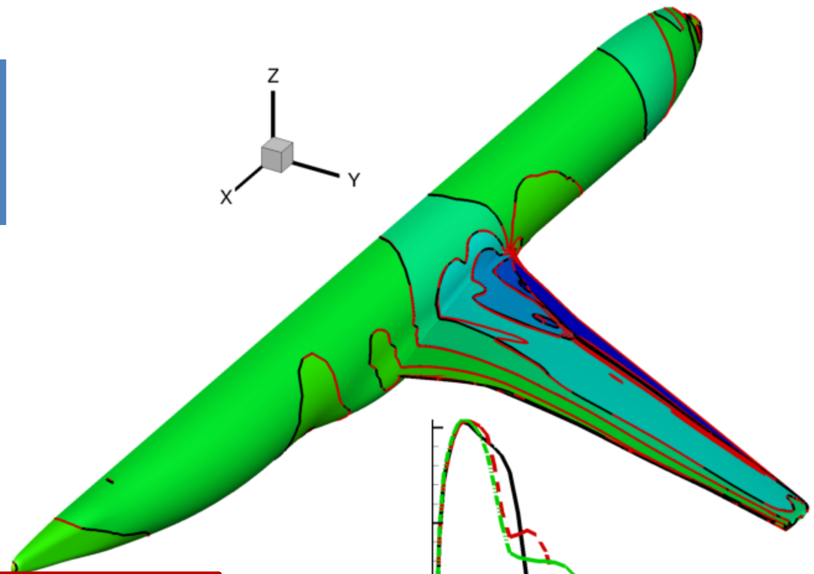
| Process | Wall-Time |
|---|---|
| ROM Building (1 core) | 1 min |
| ROM prediction (1 core) | 0.02 s |
| 1 snapshot calculation (48 cores) | 60 min |



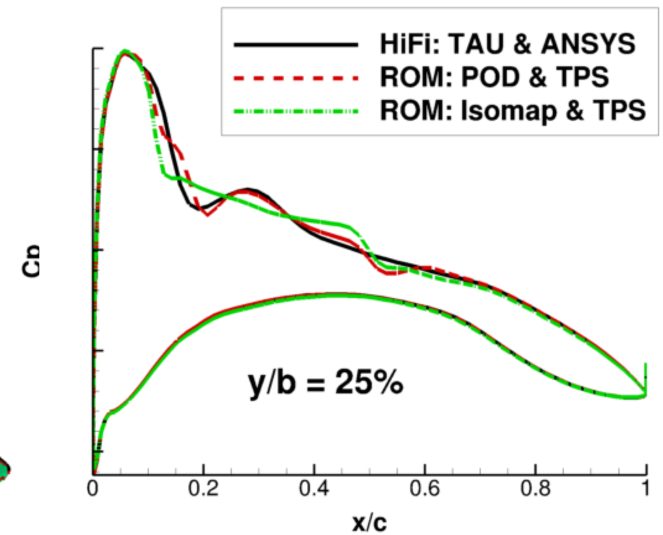

DLR

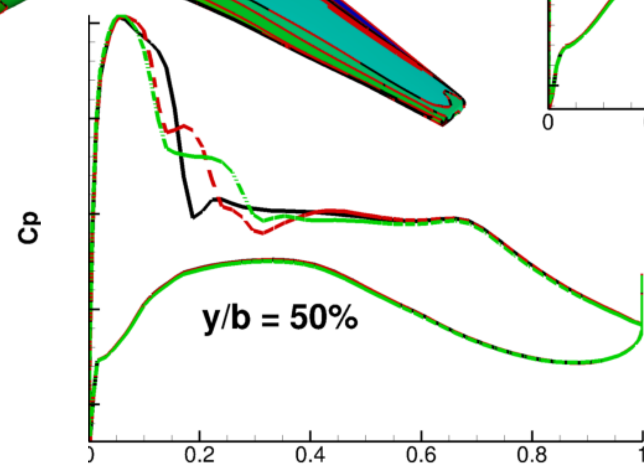
# Static aeroelastic loads prediction of the XRF1 aircraft

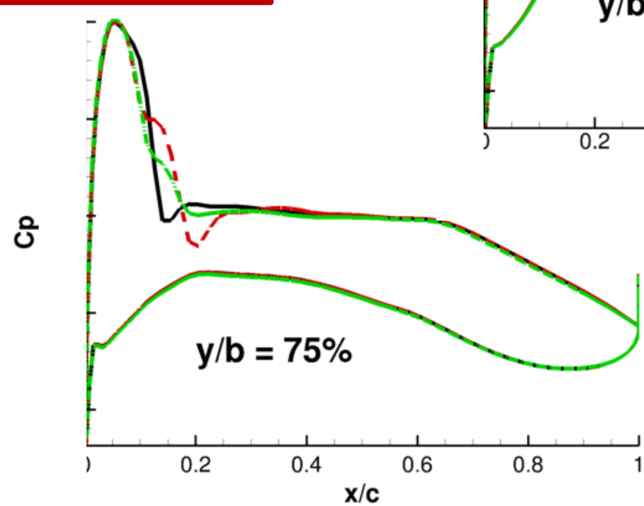
h=1000 m,
Ma=0.75,
load factor 2.5 g

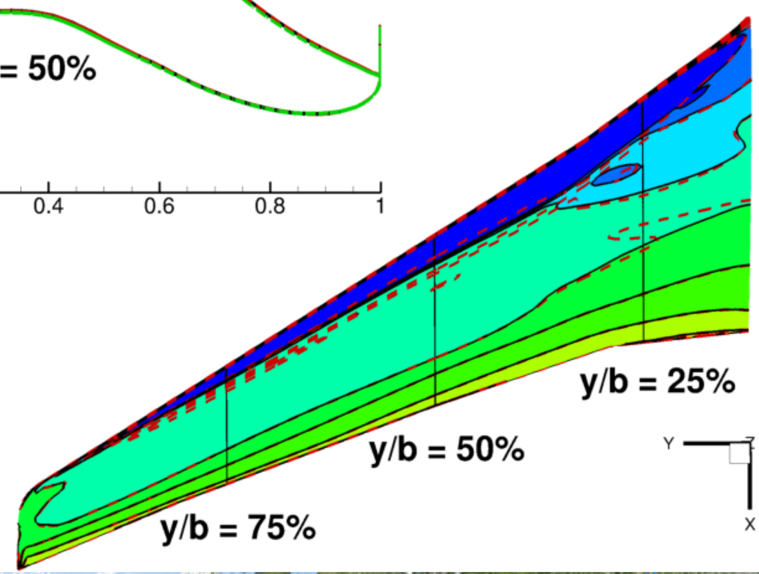



Pressure distribution visualized on the aircraft jig shape

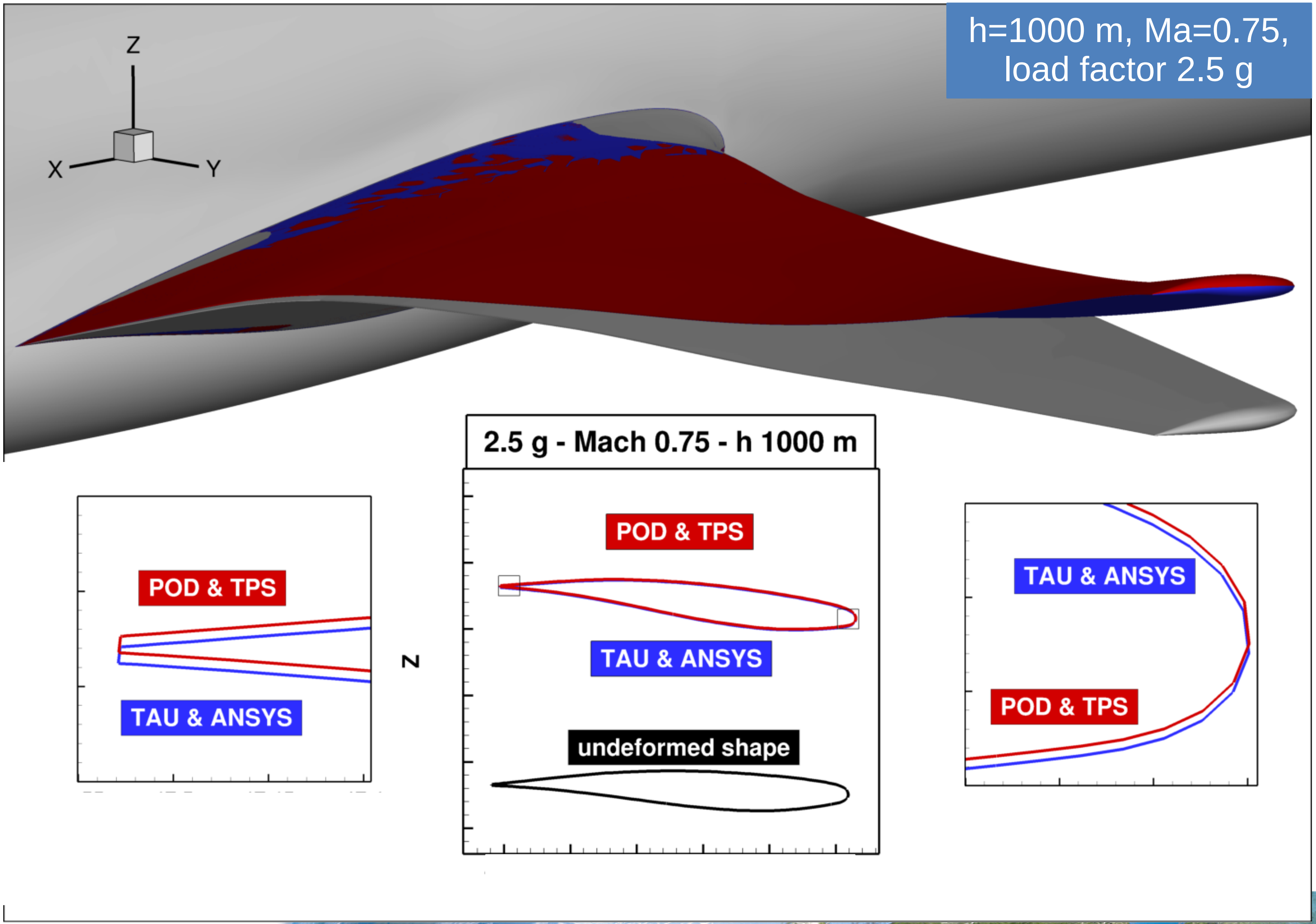


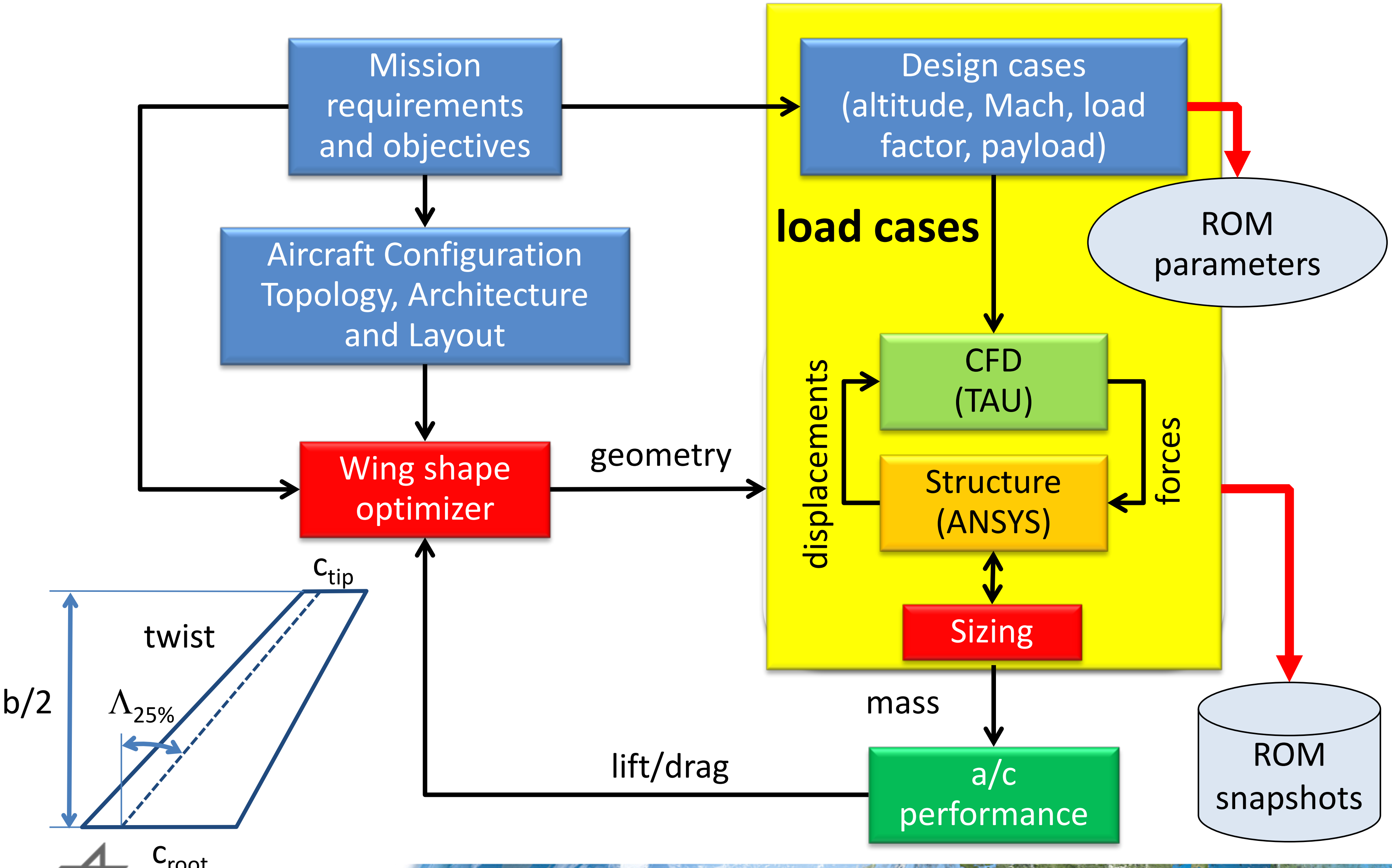



# Static aeroelastic loads prediction of the XRF1 aircraft



# MDO: static aeroelastic loads predictions using ROMs

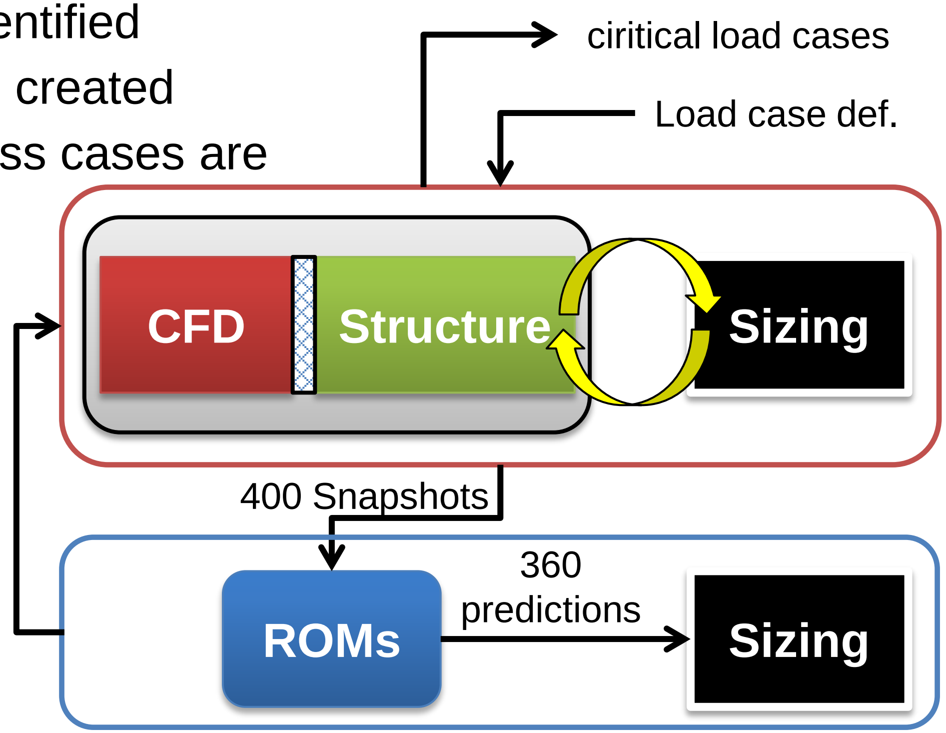


# Aeroelastic structural optimization of the XRF1 aircraft

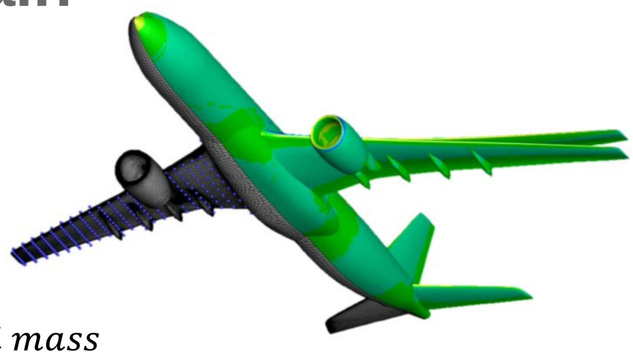
**Goal:** a priori identification of the critical load cases for 2 load factors ($-1g, 2.5g$), 5 mass cases and various altitudes/airspeeds for structural sizing

1. **400** sized aeroelastic snapshots (CFD+CSM) are computed for a discrete set of parameters ,1000 m altitude steps (0 - 12000 m)
   → 11 critical load cases are identified
2. a parametric ROM (Mach, altitude) is created
3. **360** additional predictions for two mass cases are computed using the ROM:

- 3 additional candidates for critical load cases were found
- Additional candidates for critical load cases are assessed by Hi-Fi CFD-CSM, 1 was found to be critical



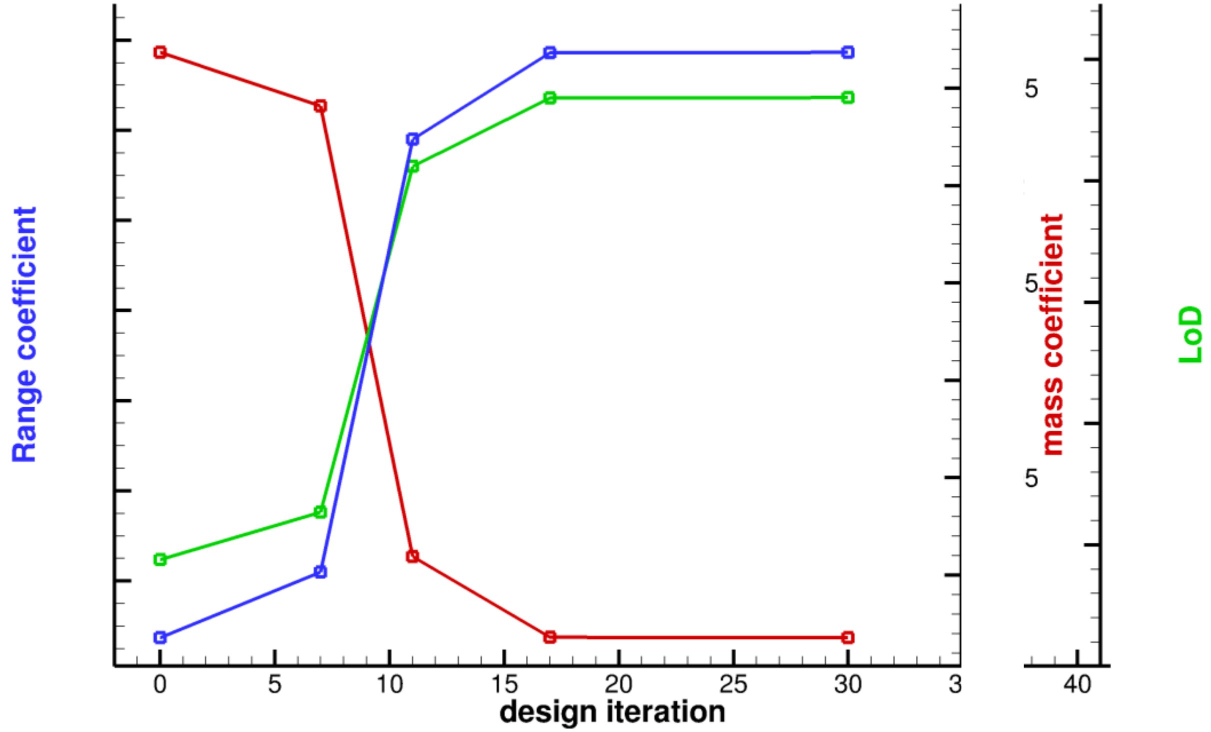
# Gradient-Based High-Fidelity MDO Chain
## Aero-Structural Optimization



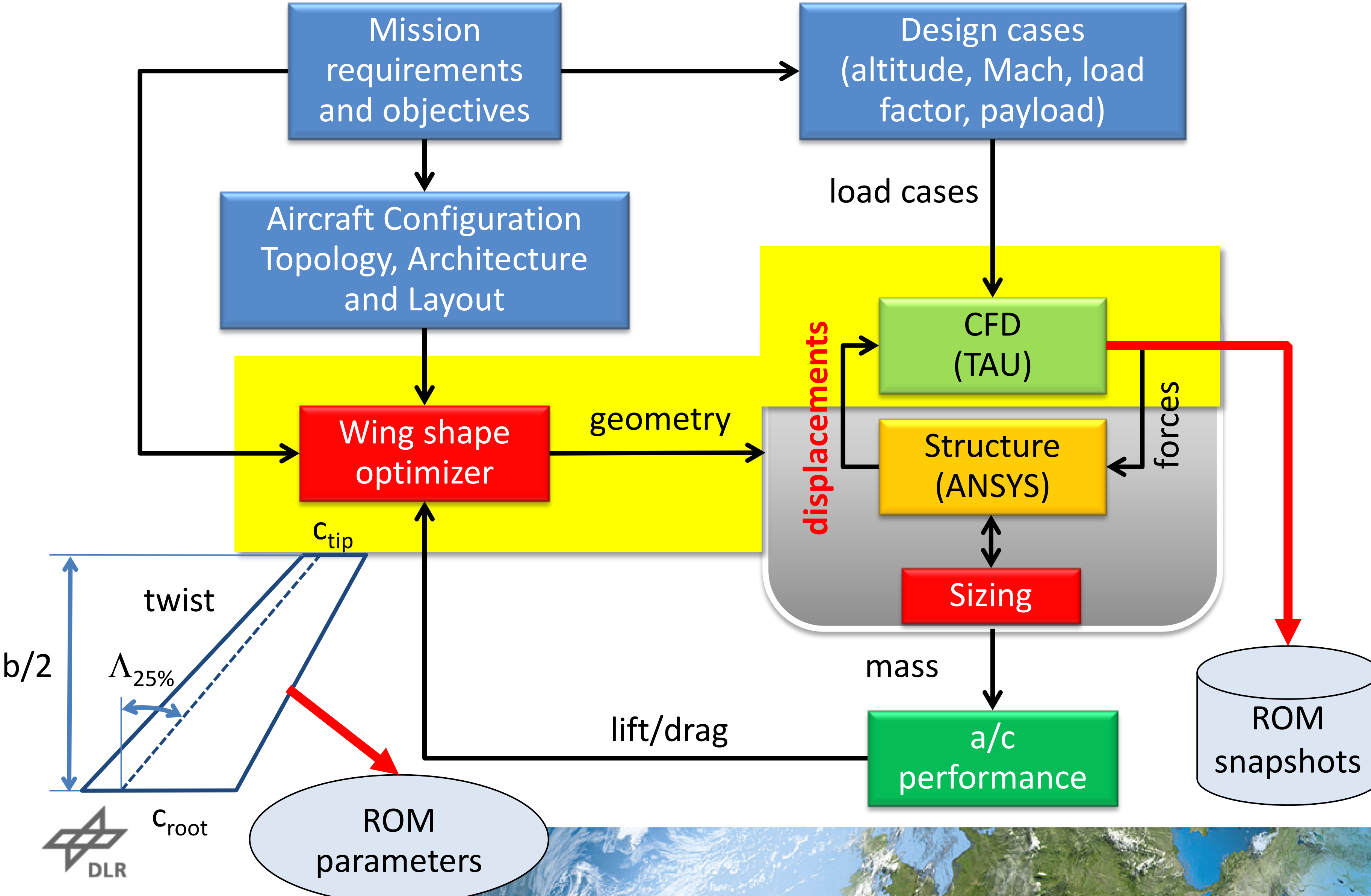
- **XRF-1 wing-body configuration**
- **Optimize the wing for the following objective:**
  - $Objective = \frac{1}{C_W} * \frac{C_L}{C_D}$, where $C_W = \frac{Current\ structural\ mass}{reference\ mass}$
- **Computed points per design iteration**:
  - **7 critical load cases** (sizing 95% of the wing) (out of 12, a priori identified)
  - **5 performance points**; 1 cruise (Ma=0.83, $C_L$=0.5) and 4 points around it (Ma±0.02, $C_L$±0.03)
- **Design variables:**
  - **360** outer shape design variables (FFD control points)
  - **348** structural thickness variables
- **Constraints:**
  - Implicit to optimizer: lift coefficient
  - Explicit to optimizer: pitching moment coefficient, strength, buckling

# Gradient-Based High-Fidelity MDO Chain
## Aero-Structural Optimization

- **Results**
  - The optimization converged after 4 optimization cycles (17 design iterations,~230 converged CFD-CSM couplings)
  - 80 hours on 96 cores were required to **increase the objective by 6%**

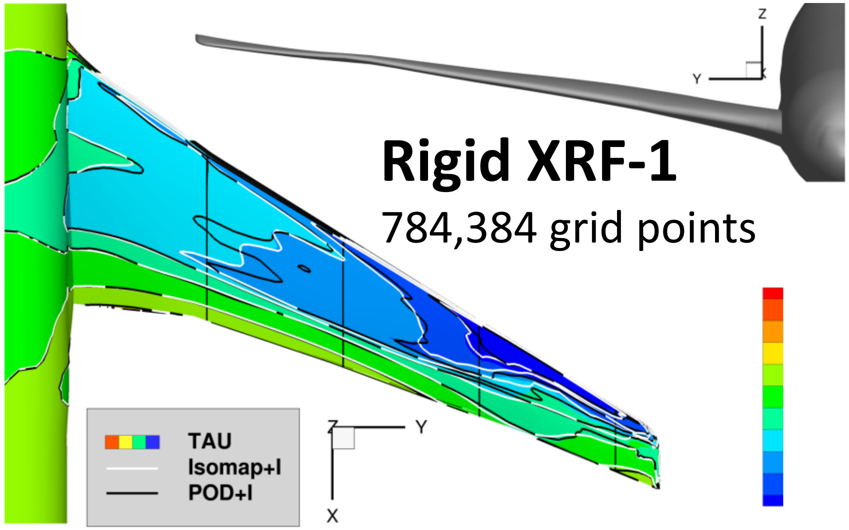


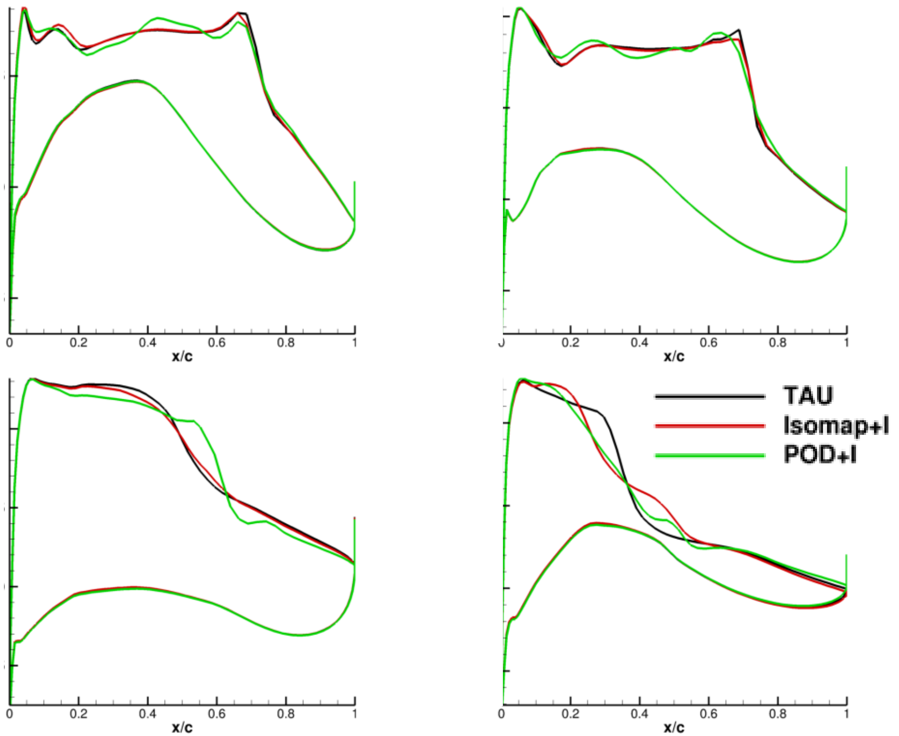
# Parametric ROM for loads and sizing in MDO



# Parametric ROM for loads and sizing in MDO

- ROM (POD+I and Isomap+I) from 100 **adaptively** sampled snapshots for $Re = 43.4 \cdot 10^6$, $Ma = 0.83$ and target $C_L = 0.5$ with TAU S-A (neg)
- 5 FFD twist parameters for the wing geometry
- **ROM for geometry variation**
- Predictions point: largest "gap" between snapshots in parameter space.

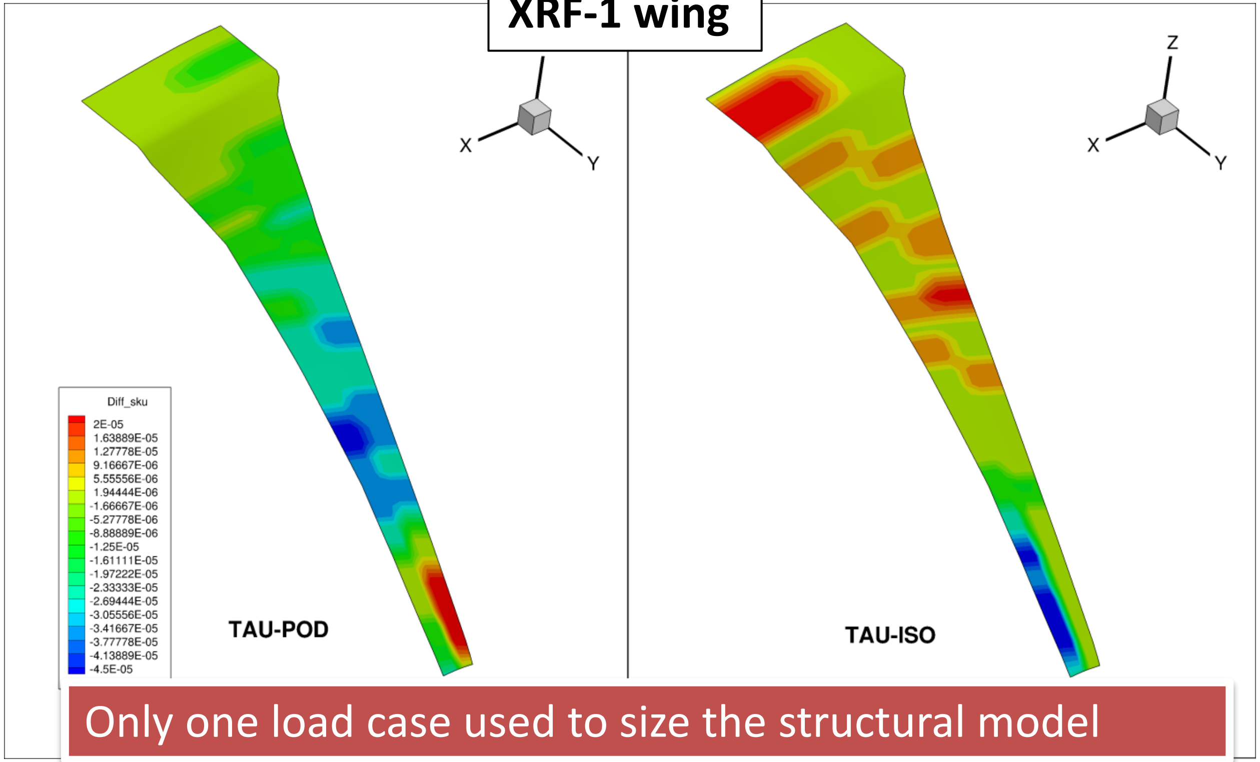


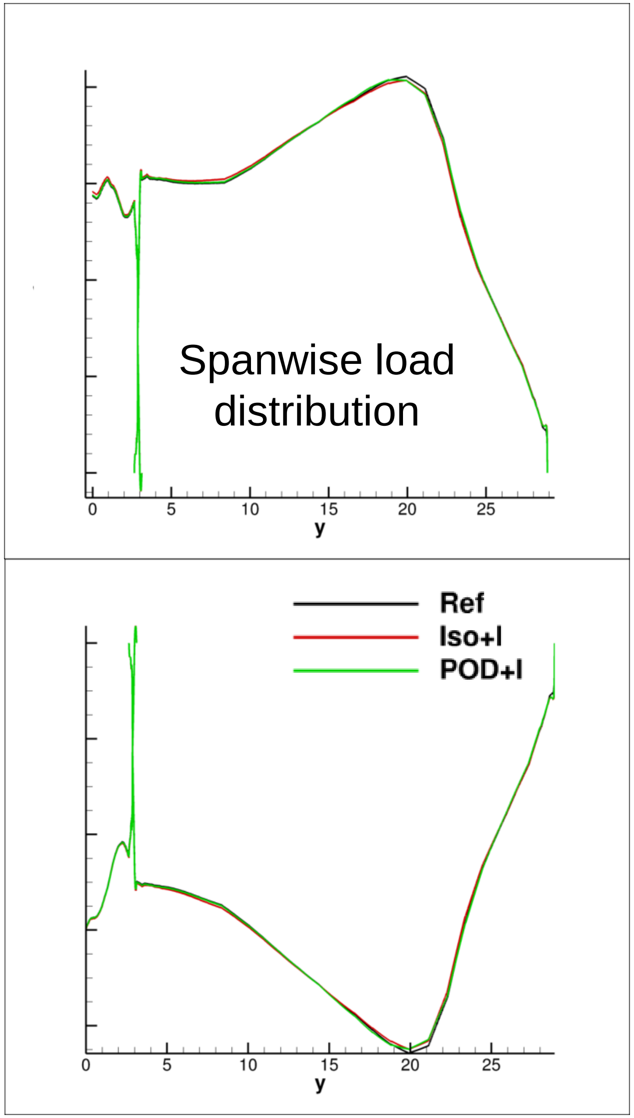




| Process | Wall-Time |
|---|---|
| Isomap ROM building (1 core) | 2 min |
| POD ROM building (1core) | 0.2 s |
| ROM prediction (1 core) | 0.02 s |
| 1 snapshot calculation (24 cores) | 70 min |

# Parametric ROM for loads and sizing in MDO

XRF-1 wing

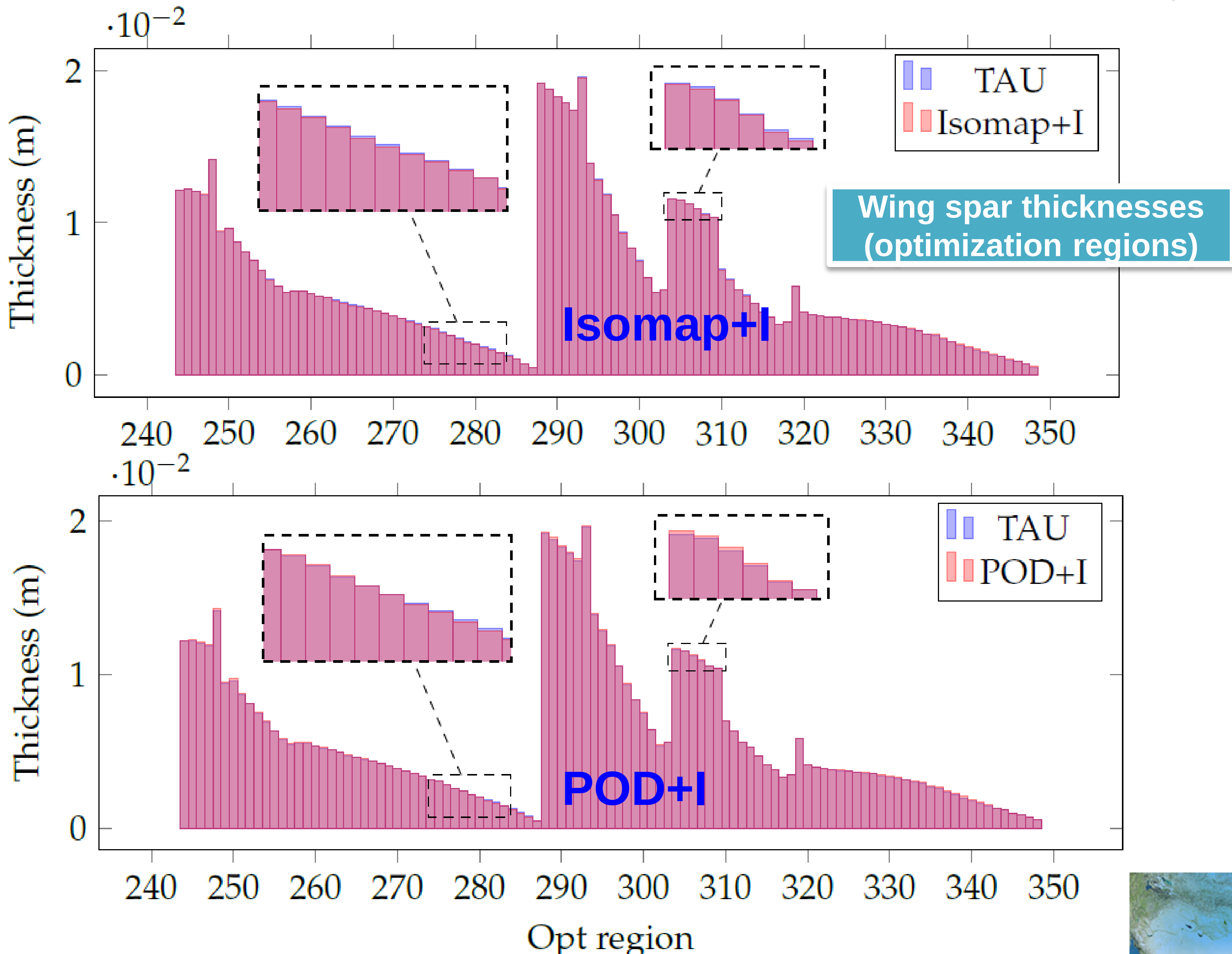



**Only one load case used to size the structural model**

| Method | Wing mass | Difference |
|---|---|---|
| CFD (TAU) ref. | 3761.1 kg | - |
| ROM POD+I | 3776.1 kg | +0.4% |
| ROM Isomap + I | 3755.8 kg | -0.1% |

# Parametric ROM for loads and sizing in MDO



# Outline

1. Motivation and objectives
2. Intro to our favorite Reduced-Order Modeling (ROM) methods
3. Applications:
   - multi-disciplinary optimization (MDO)
     - static aeroelastic predictions
     - loads prediction and structural sizing
     - efficient selection of critical load cases
   - **unsteady aerodynamics and maneuvers**
4. Summary & outlook

# Nonlinear unsteady LSQ-ROM approach for aerodynamic applications

**Semi-discrete unsteady Navier-Stokes Eqs.** $\widehat{\mathbf{R}} \stackrel{\text{def}}{=} \mathbf{R}\big(\mathbf{w}(\mathrm{t})\big) + \boldsymbol{\Omega}\frac{\partial \mathbf{w}(\mathrm{t})}{\partial \mathrm{t}} = \mathbf{0} \in \mathbb{R}^N$ $\boldsymbol{\Omega}$: cell volumes

- **N**: order of CFD model (variables x nodes)
- **r**: order of the ROM (i.e. number of POD modes)

Search for an approximated solution $\mathbf{w} = [\rho, \rho\mathbf{v}, \rho E^t, \nu_t] \in \mathbb{R}^N$

- in the POD subspace $\mathbf{U}_r \in \mathbb{R}^{N \times r}, r \ll N$
- minimizing a the **unsteady residual** in the $\mathrm{L}_2$ norm

$$\mathbf{w} \approx \sum_{i=1}^{r} a_i \mathbf{U}^i + \overline{\mathbf{w}} = \mathbf{U}_r \mathbf{a} + \overline{\mathbf{w}}$$

$\mathbf{a}$: vector of the unknown coefficients $a_i$

$\overline{\mathbf{w}}$: mean of the snapshots set

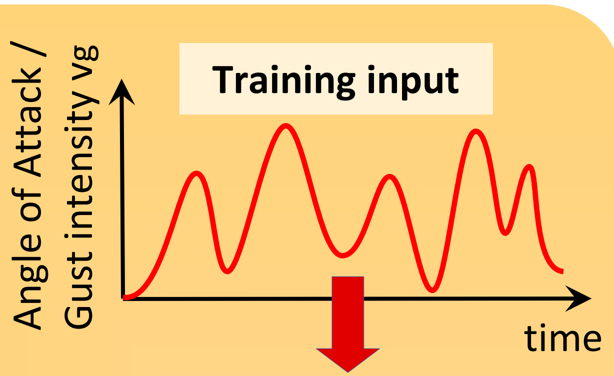
$$\min_{\mathbf{a}} \| \widehat{\mathbf{R}}(\mathbf{U}_r\mathbf{a} + \overline{\mathbf{w}}) \|_{\mathrm{L}_2}^2$$

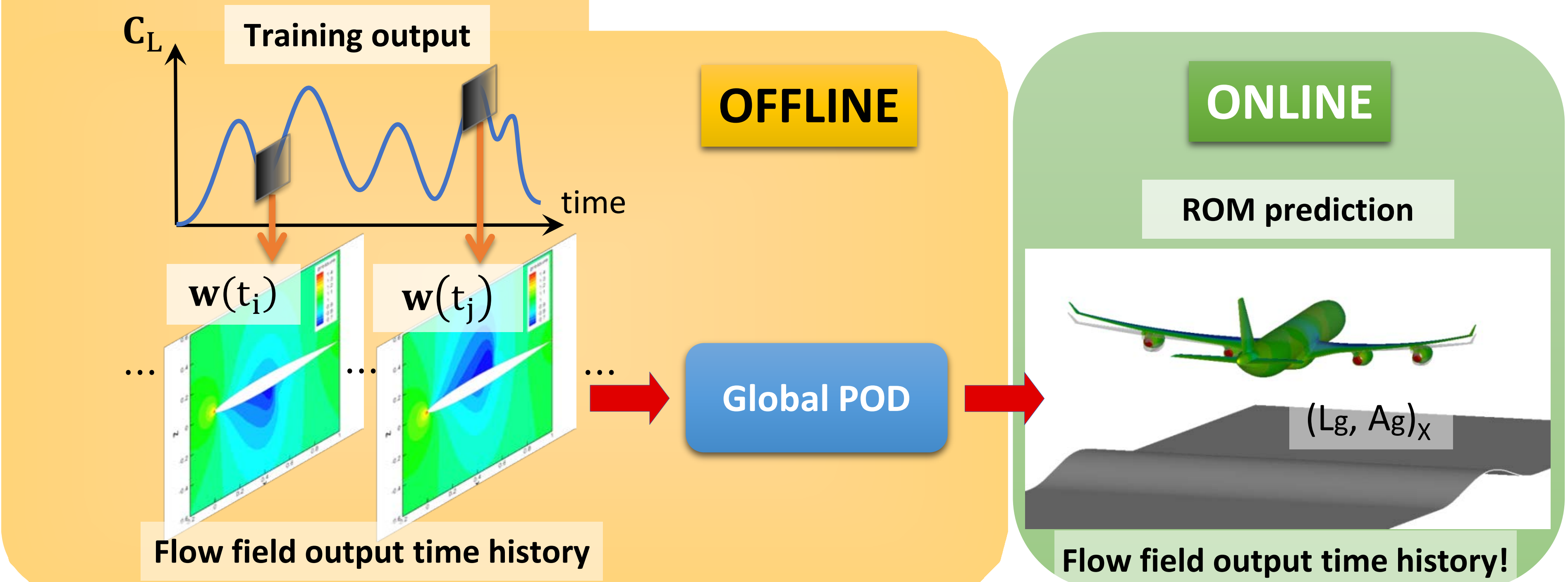
nonlinear least squares problem

# Building the POD subspace

Collecting snapshots coming from an unsteady simulation
→ variation in the local effective **angle of attack**

- Boundary conditions disturbances
- Motion (forced or induced)
- Gust perturbations (here: rigid aircraft, no motion)



# Model order reduction for unsteady CFD

Per each physical time step, find the POD coefficients minimizing the unsteady residual at the time step *n+1,* by solving a nonlinear least squares problem (L-M algo.):

until convergence

$$\mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n+1}}) \approx \mathbf{U}\mathbf{a} + \overline{\mathbf{w}}$$

$$\widehat{\mathbf{R}} \stackrel{\mathrm{def}}{=} \mathbf{R}(\mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n+1}})) + \boldsymbol{\Omega}\frac{3\mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n+1}}) - 4\mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n}}) + \mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n-1}})}{2\Delta\mathrm{t}}$$

$$(\mathbf{J}^{\mathrm{T}}\mathbf{J} + \boldsymbol{\lambda}\mathbf{I})\Delta\mathbf{a} = -\mathbf{J}^{\mathrm{T}}\widehat{\mathbf{R}}$$

$$\mathbf{a} \leftarrow \mathbf{a} + \Delta\mathbf{a}$$

$\mathbf{J} \in \mathbb{R}^{\boldsymbol{N}\times\boldsymbol{r}}$

- **N**: order of CFD model (variables x nodes)
- **r**: order of the ROM (i.e. number of POD modes)

**J** is initialized at each time step via finite differences, or kept equal to those of the previous time step if $\left\|\mathbf{J}^{\mathrm{T}}\widehat{\mathbf{R}}\right\| < \epsilon$

# Model order reduction for unsteady CFD

Per each physical time step, find the POD coefficients minimizing the unsteady residual at the time step *n+1*, by solving a nonlinear least squares problem (L-M algo.):

until convergence

$$\mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n+1}}) \approx \mathbf{Ua} + \overline{\mathbf{w}}$$

$$\widehat{\mathbf{R}} \stackrel{\mathrm{def}}{=} \mathbf{R}(\mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n+1}})) + \mathbf{\Omega} \frac{3\mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n+1}}) - 4\mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n}}) + \mathbf{w}_{\mathrm{rom}}(\mathrm{t}_{\mathrm{n-1}})}{2\Delta\mathrm{t}}$$

$$(\mathbf{J}^{\mathrm{T}}\mathbf{J} + \boldsymbol{\lambda}\mathbf{I})\Delta\mathbf{a} = -\mathbf{J}^{\mathrm{T}}\widehat{\mathbf{R}}$$

$$\mathbf{J} \in \mathbb{R}^{N \times r}$$

- **N**: order of CFD model (variables x nodes)
- **r**: order of the ROM (i.e. number of POD modes)

Broyden's method for updating **J**

$$\mathbf{J}_{k+1} = \mathbf{J}_k + \frac{\Delta\widehat{\mathbf{R}} - \mathbf{J}_k\,\Delta\mathbf{a}}{\|\Delta\mathbf{a}\|^2}\Delta\mathbf{a}^{\mathrm{T}}$$

Updating pseudo-Hessian matrix $\mathbf{A} \equiv \mathbf{J}^{\mathrm{T}}\mathbf{J} \in \mathbb{R}^{r \times r} \Rightarrow \mathcal{O}(\mathrm{Nr}^2)$

$$\mathbf{A}_{k+1} \equiv \mathbf{J}_{k+1}^{\mathrm{T}}\mathbf{J}_{k+1}$$

$$= \mathbf{A}_k + \frac{\mathbf{J}_k^{\mathrm{T}}\,\Delta\widehat{\mathbf{R}} - \mathbf{A}_k\,\Delta\mathbf{a}}{\|\Delta\mathbf{a}\|^2}\Delta\mathbf{a}^{\mathrm{T}} + \Delta\mathbf{a}\frac{\Delta\widehat{\mathbf{R}}^{\mathrm{T}}\,\mathbf{J}_k - \Delta\mathbf{a}^{\mathrm{T}}\mathbf{A}_k}{\|\Delta\mathbf{a}\|^2}$$

$$+ \Delta\mathbf{a}\frac{\Delta\widehat{\mathbf{R}}^{\mathrm{T}}\Delta\widehat{\mathbf{R}} - \Delta\widehat{\mathbf{R}}^{\mathrm{T}}\mathbf{J}_k\,\Delta\mathbf{a} - \Delta\mathbf{a}^{\mathrm{T}}\mathbf{J}_k^{\mathrm{T}}\,\Delta\widehat{\mathbf{R}} + \Delta\mathbf{a}^{\mathrm{T}}\mathbf{A}_k\Delta\mathbf{a}}{\|\Delta\mathbf{a}\|^4}\Delta\mathbf{a}^{\mathrm{T}} \quad \Rightarrow \mathcal{O}(\mathrm{Nr})$$

$$\mathbf{a} \leftarrow \mathbf{a} + \Delta\mathbf{a}$$

# Model order reduction for unsteady CFD

## Curse of dimensionality in ROMs of nonlinear systems

$\min_{\mathbf{a}} \| \hat{\mathbf{R}}(\mathbf{U}\mathbf{a} + \overline{\mathbf{w}}) \|_{L_2}^2$ → nonlinear least squares problem

Jacobian Matrix $\mathbf{J} \in \mathbb{R}^{N \times r}$

- **N**: order of CFD model (variables x nodes)
- **r**: order of the ROM (i.e. number of POD modes)

I. compute the approx. solution

$$\mathbf{w}(\mathrm{t}) \approx \mathbf{U}\,\mathbf{a}(\mathrm{t}) + \overline{\mathbf{w}} \quad \Rightarrow \mathcal{O}(\mathrm{Nr})$$

II. evaluate the unsteady residual

$$\hat{\mathbf{R}}\big(\mathbf{w}(\mathrm{t})\big) \stackrel{\text{def}}{=} \mathbf{R}\big(\mathbf{w}(\mathrm{t})\big) + \mathbf{\Omega}\frac{\partial \mathbf{w}(\mathrm{t})}{\partial \mathrm{t}} \Rightarrow \mathcal{O}(\mathrm{N})$$

III. solve the LS problem

$$\left(\mathbf{J}^{\mathrm{T}}\mathbf{J} + \lambda\mathbf{I}\right)\Delta\mathbf{a} = -\mathbf{J}^{\mathrm{T}}\hat{\mathbf{R}}$$

→

i. Compute RHS $-\mathbf{J}^{\mathrm{T}}\hat{\mathbf{R}}$

ii. Broyden's method for updating $\mathbf{J}$

iii. Updating pseudo-Hessian matrix $\mathbf{A} \equiv \mathbf{J}^{\mathrm{T}}\mathbf{J} \in \mathbb{R}^{r \times r}$

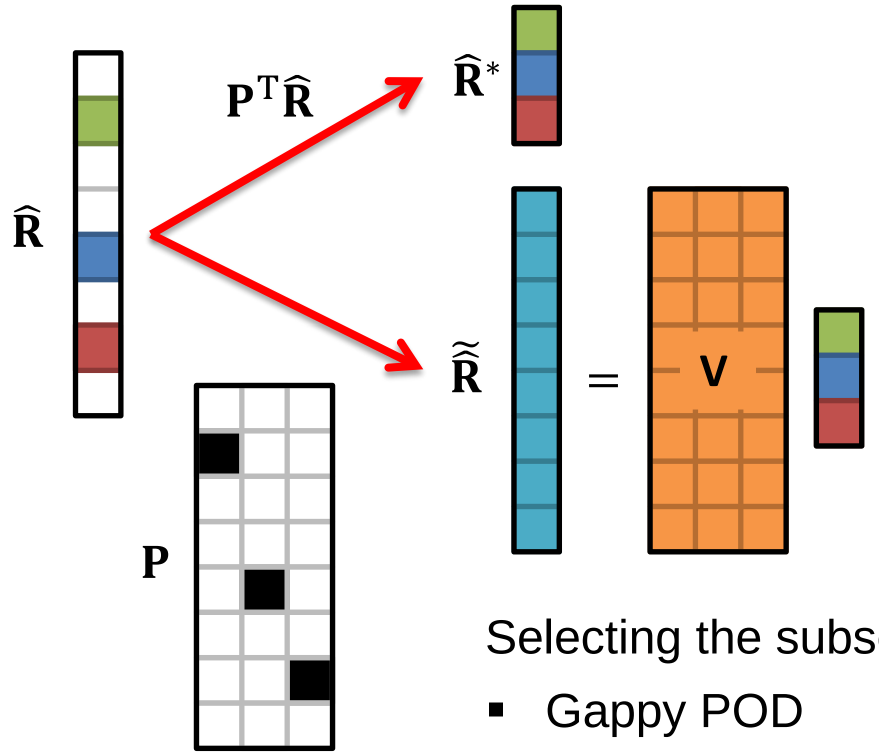
$\Rightarrow \mathcal{O}(\mathrm{Nr})$

The computational cost scales linearly with the dimension **N** of the full order model. No significant speedup can be expected when solving the minimum residual ROM.

DLR

# Hyper-reduction approaches

Complexity reduction by sampling (or compute only a few entries of) the nonlinear unsteady residual vector $\hat{\mathbf{R}}$




mask matrix

**Collocation**
- omission of many components
- non intrusive

**Reconstruction**
- approximation of the entire vector, by interpolation or by least-squares projection onto a subspace $\mathbf{V} = \mathbf{U}(\mathbf{U}^T\mathbf{P}\,\mathbf{P}^T\mathbf{U})^{-1}\mathbf{U}^T\mathbf{P}$

Selecting the subset indices →
- Gappy POD
- Missing Point Estimation
- (Discrete) Empirical Interpolation Method

**Greedy:** minimize

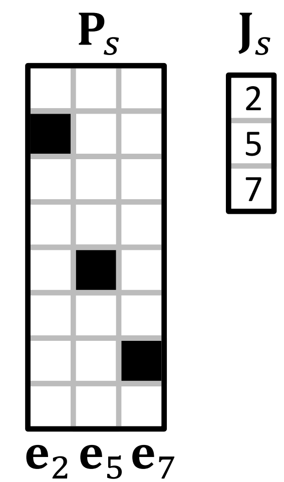
$$\|\mathbf{U}(\mathbf{U}^T\mathbf{P}\,\mathbf{P}^T\mathbf{U})^{-1}\mathbf{U}^T\mathbf{P}\,\mathbf{P}^T\| = 1/\sigma_{\min}(\mathbf{P}^T\mathbf{U})$$

- The complete nonlinear unsteady residual vector $\hat{\mathbf{R}}$ is evaluated,
- but only a small subset of its entries are used in the minimization process

# Exhaustive greedy missing point estimation procedure

Greedy point selection algorithms minimize an error indicator by sequentially **looping over all entries** → costly ⇒ > $\mathcal{O}(\mathrm{Nr}^3)$

Exhaustive greedy MPE for maximizing $\sigma_{\min}(\mathbf{P}_{s+1}^T\mathbf{U})$




**Input**: $\mathbf{U} \in \mathbb{R}^{N\times r}$: basis of a r-dim subspace,
$\mathbf{J}_s \in \mathbb{R}^{s\times 1}$: index set,
$\mathbf{P}_s \in \mathbb{R}^{N\times s}$: mask matrix with s indices, where s ≥ r.

1. $\sigma_{\text{opt}} = 0,\ \bar{\mathbf{J}}_s = \{1,\dots,N\} \setminus \mathbf{J}_s$
2. **for** $j \in \bar{\mathbf{J}}_s$ **do**
3. $\widetilde{\mathbf{P}} = \left(\mathbf{P}_s, \mathbf{e}_j\right) \in \mathbb{R}^{N\times(s+1)}$
4. Compute $\sigma_{\min}\left(\widetilde{\mathbf{P}}^T\mathbf{U}\right)$
5. **if** $\sigma_{\text{j}} > \sigma_{\text{opt}}$ **then**
6. $\sigma_{\text{opt}} = \sigma_{\text{j}}\,;\ j_{\text{opt}} = j$

Singular Value Decomposition

rank-one SVD update

**Output**: next index $\mathbf{J}_{s+1} = \mathbf{J}_s \cup \{j_{\text{opt}}\}$, $\mathbf{P}_{s+1}^T = \left[\mathbf{P}_s, \mathbf{e}_{j,\,\text{opt}}\right]$

The penultimate singular value bounds the growth of $\sigma_{\min}$

# Accelerated greedy missing point estimation procedure

Rank-one SVD update

$$\mathbf{U}^T\mathbf{P}_{s+1}\mathbf{P}_{s+1}^T\mathbf{U} = \boldsymbol{\Phi}_s\left(\boldsymbol{\Sigma}_s^2 + \mathbf{v}_j\mathbf{v}_j^T\right)\boldsymbol{\Phi}_s^T$$

$$\mathbf{v}_j := \left(\mathbf{e}_{j,\,s+1}^T\mathbf{U}\boldsymbol{\Phi}_s\right)^T$$

Symmetric rank-one eigenvalue modification

Eigenvalues

$$\lambda_i = \sigma_i^2 + \mu_i\|\mathbf{v}\|^2$$

$$0 < \mu_i < 1$$

$$\sum_{i=1}^r \mu_i = 1$$

**Idea:**

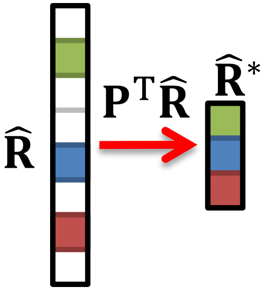
- Select the vector $\mathbf{v}_j$ leading to the largest growth in the smallest eigenvalue $\lambda_{\min}(\mathbf{v}_j)$ of $\mathbf{M} := \boldsymbol{\Sigma}_s^2 + \mathbf{v}_j\mathbf{v}_j^T \in \mathbb{R}^{r\times r}$
- Build fast approximations that sort the set of candidate vectors that induce the rank-one modifications (→ without solving the modified eigenvalue pb.)

Select the vectors $\mathbf{v}_{j,\,\mathrm{opt}}$ that feature the largest absolute values **in the ultimate component** while all other components are comparably small.

R. Zimmermann and K. Willcox. **An accelerated greedy missing point estimation procedure**. *SIAM Journal on Scientific Computing*, 38(5), A2827–A2850. DOI:10.1137/15M1042899

# Accelerated greedy MPE with rank-1 SVD update

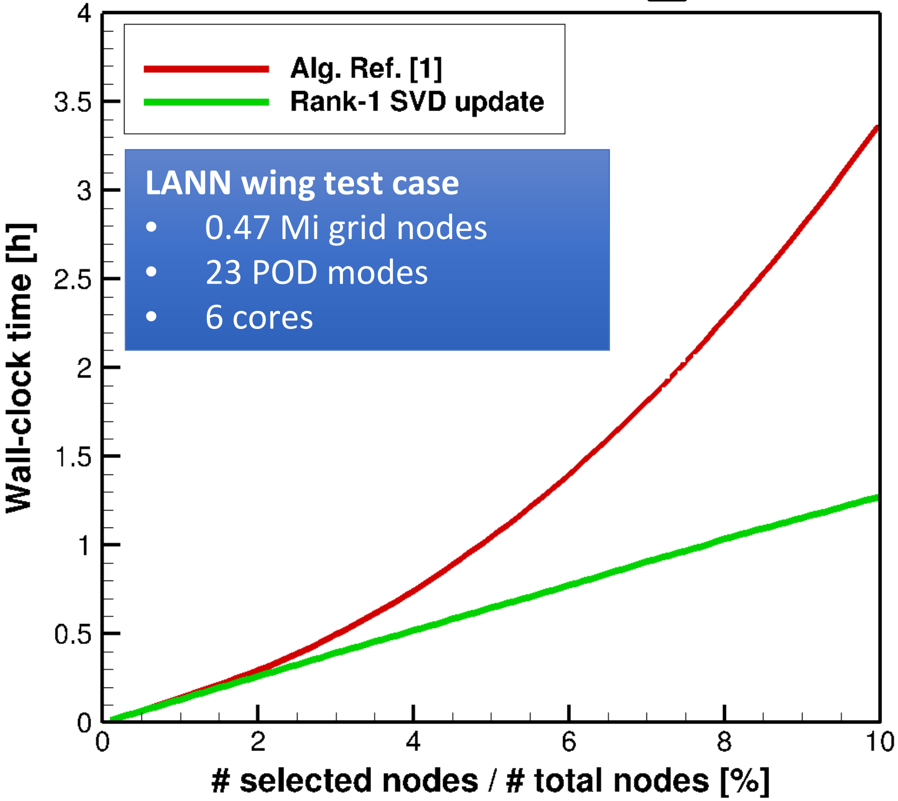
Using an (additional) **rank-1 SVD update** within the iterative greedy step, to further accelerate the selection of the grid nodes




**Computational cost**

- Alg. Ref. [1]: $\mathcal{O}\left(\mathbf{N}\mathbf{r}^2\mathbf{s} + \mathbf{r}\,\mathbf{s}^3\right)$
- Rank-1 SVD update: $\mathcal{O}\left(\mathbf{N}\mathbf{r}^2\mathbf{s} + \mathbf{r}^3\mathbf{s}\right)$

- **N**: order of CFD model (variables x nodes)
- **r**: order of the ROM (i.e. number of POD modes)
- **s** (> **r**): number of MPE selected nodes

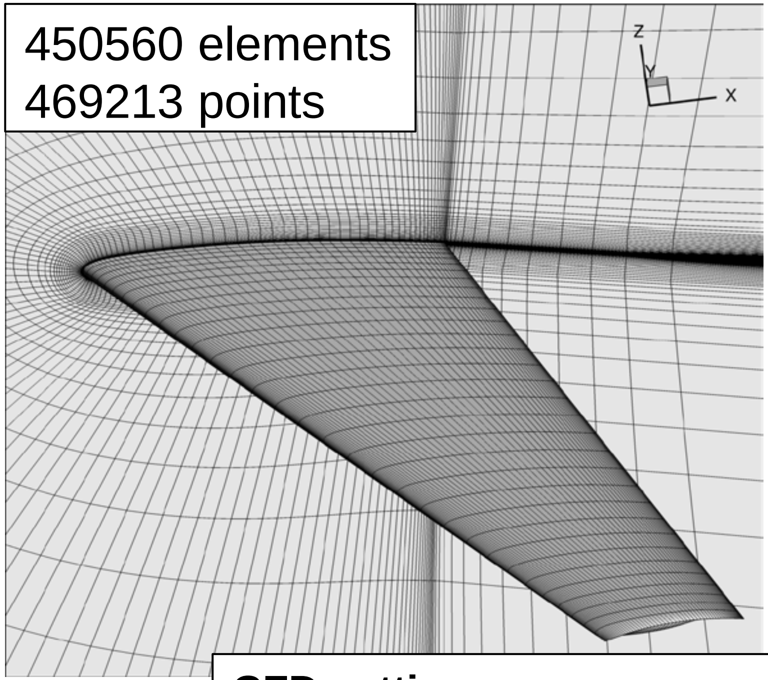


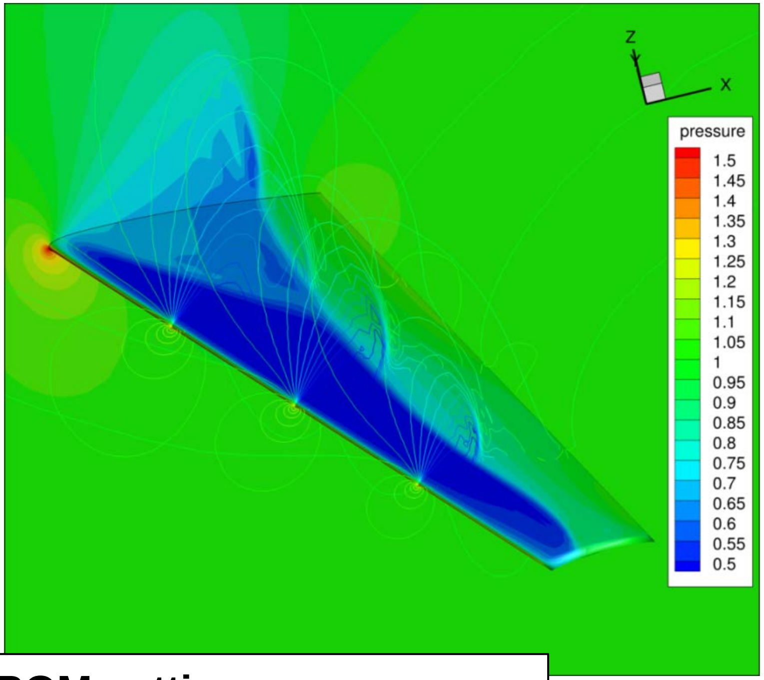

[1] R. Zimmermann and K. Willcox. **An accelerated greedy missing point estimation procedure**. *SIAM Journal on Scientific Computing*, 38(5), A2827–A2850. DOI:10.1137/15M1042899

# Unsteady loads prediction for the LANN wing

- RANS equations with SA-neg turbulence model
- $V_\infty$ = 271.66 m/s, Mach = 0.82, Re = 7.17 $10^6$
- Chirp training maneuver exciting up to $\mathrm{k} \stackrel{\text{def}}{=} \frac{\omega c_r}{V_\infty} = 0.35$
- Predicted periodic pitching oscillation: $\alpha(\tau) = 2.6° + 0.25\sin(\mathbf{k}\,\tau)$, $\tau \stackrel{\text{def}}{=} \frac{V_\infty t}{c_r}$

450560 elements
469213 points



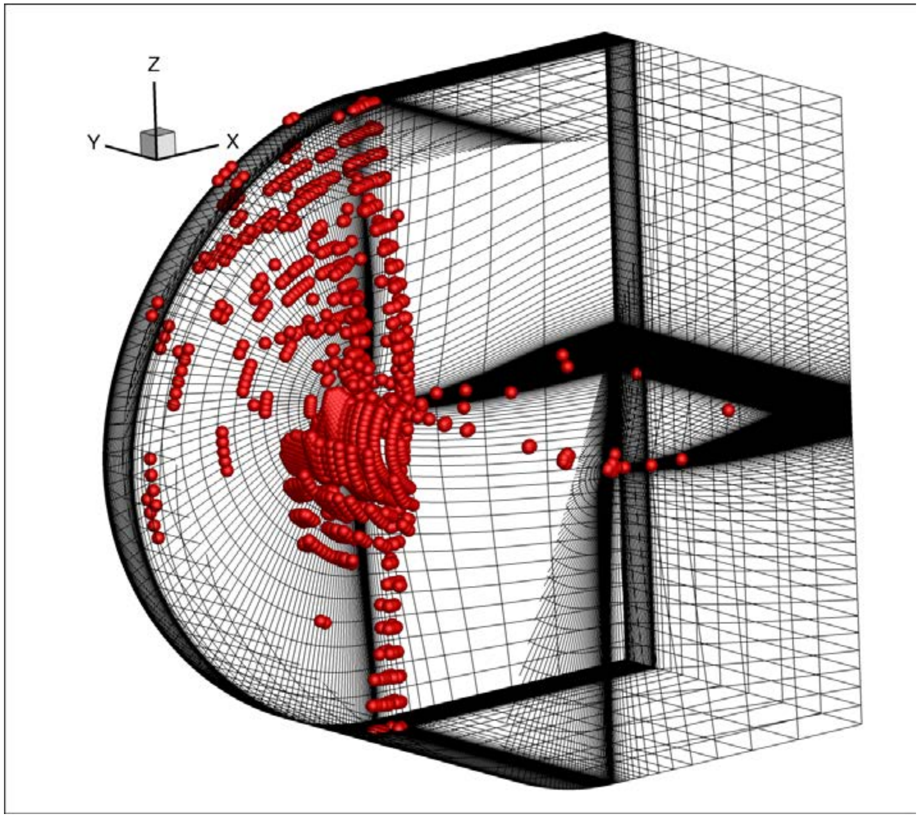

**CFD settings**
- Min residual: 1e-5
- Max inner iterations: 100

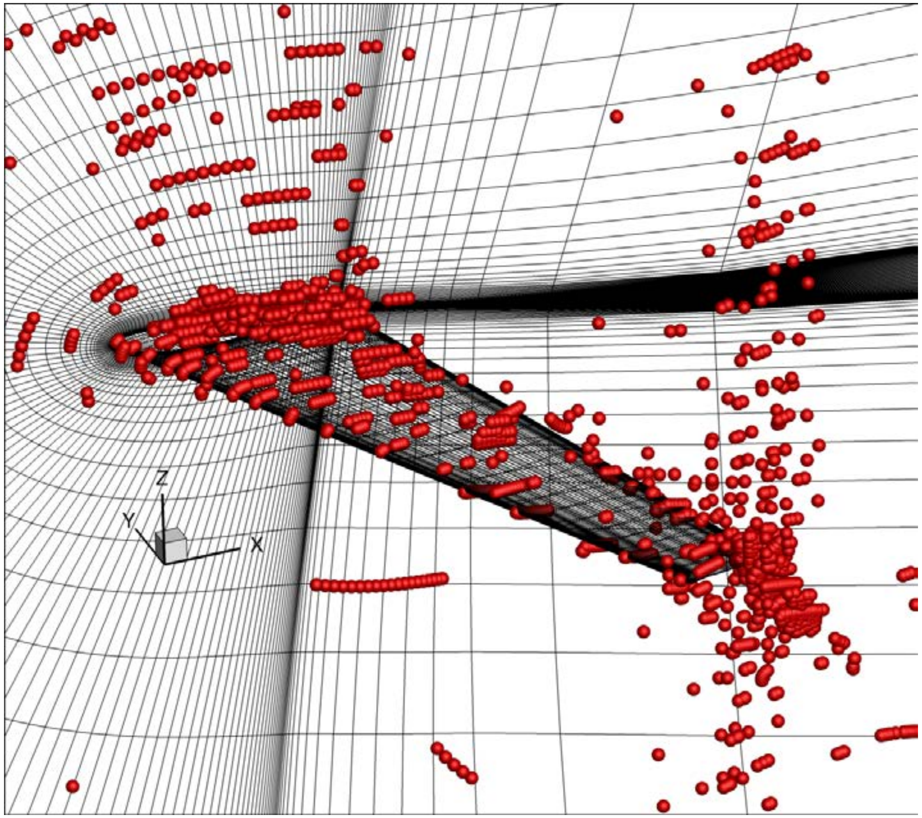
**ROM settings**
- 18 POD modes
- 5000 MPE nodes

DLR

# Unsteady loads prediction for the LANN wing

Total number of grid nodes: **469213**



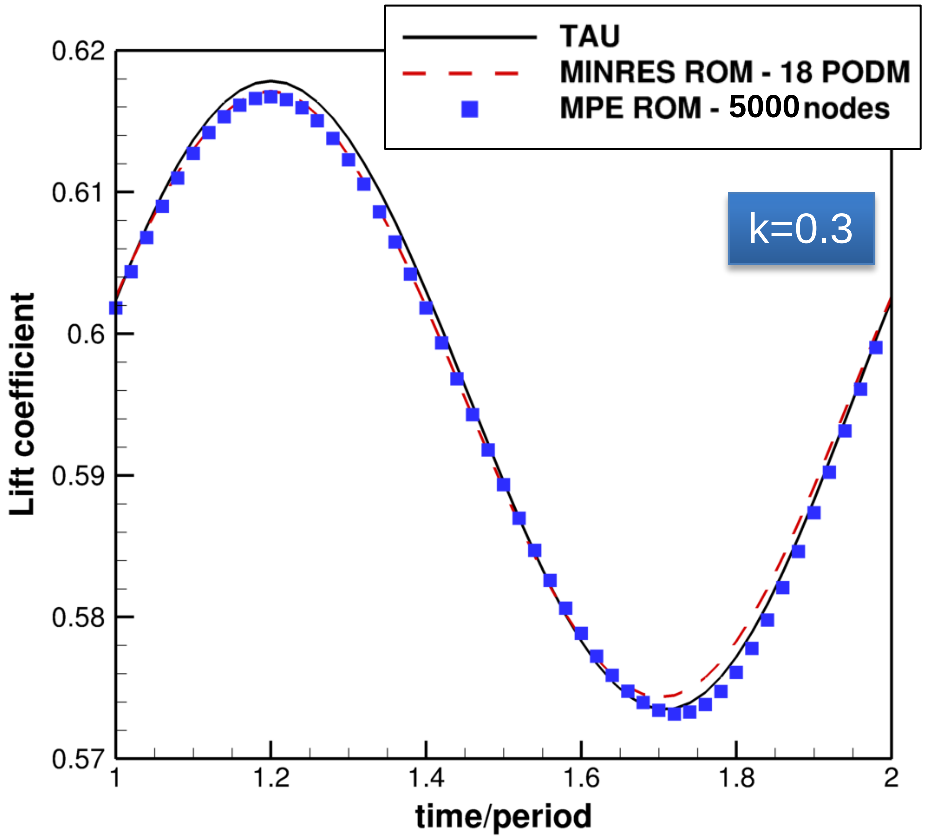



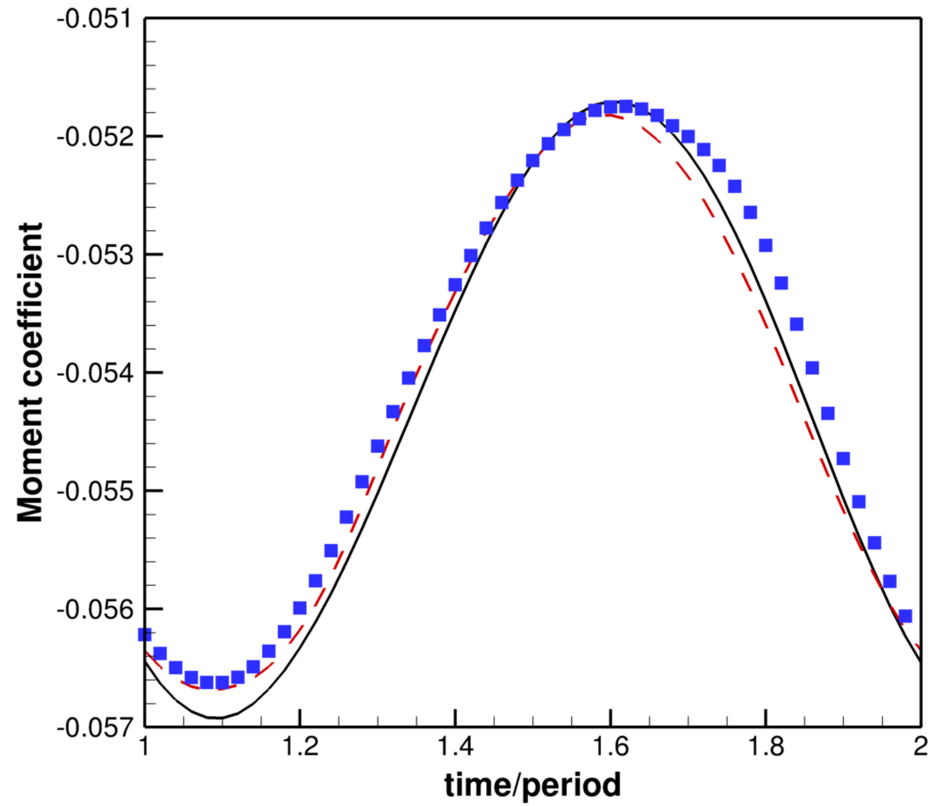

**5000 nodes** selected by the greedy MPE

~1% of the total number of grid nodes

DLR

# Unsteady loads prediction for the LANN wing




**Offline Performances (10 cores)**

**Compute snapshots**: 4d 10h 40min
**POD ROM building**: ~5 min
**Fast greedy MPE**: ~6 min

| Online Performances | | CFD (10 cores) | ROM (10 cores) | |
|---|---|---|---|---|
| | | | w/o MPE | with MPE |
| Average Wall Time | total | 25 h 50 min | 2 h 44 min | 1 h 29 min |
| | per time step | ~16 min | 1 min 36 s | 36 s |
| | speed-up | 1 | 9.5 | **26.7** |

# Unsteady ROM prediction assessment for a full aircraft

$$k \stackrel{\text{def}}{=} \frac{\omega c_r}{V_\infty} \qquad \tau \stackrel{\text{def}}{=} \frac{V_\infty t}{c_r}$$

- RANS equations with SA-neg turbulence model
- $V_\infty$ = 246 m/s, Mach = 0.83, Re = 6.5 $10^6$
- Linear chirp training maneuver exciting up to $k_{max} = 3$:

$$\alpha(\tau) = 2.0164° + 2.3266 \sin(k\ \tau)$$

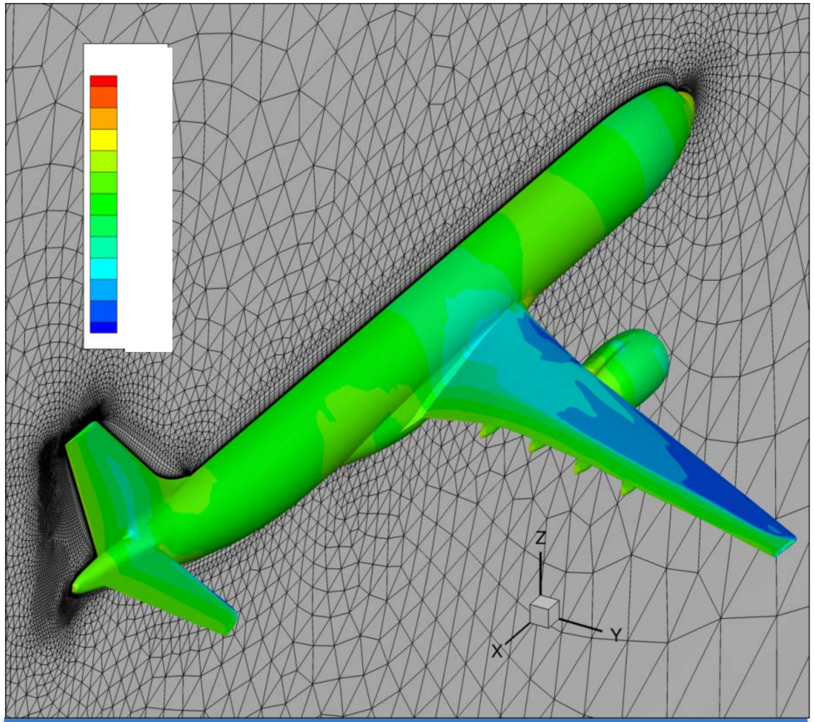
with $k(\tau) = k_{max} \frac{c_r}{V_\infty} \tau$

- Predicted (1-cos)–like pitching oscillation at $k = 0.33$

**CFD setting**
- Dual time stepping
- Min residual: 1e-6
- Max inner iterations per time step: 5000
- Physical time steps: 550
- Linearly distributed time steps

**ROM setting**

Truncation of the POD modes to 99.999% of their energy content (~100 modes)

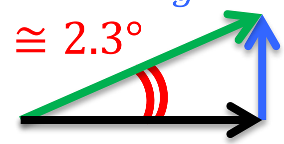



Steady state

# Unsteady ROM prediction assessment for a full aircraft

$\boldsymbol{w_g} = 10\ m/s$

$\Delta\alpha \cong 2.3°$



$\boldsymbol{V_\infty} = 246\ m/s$

**Forced Motion**

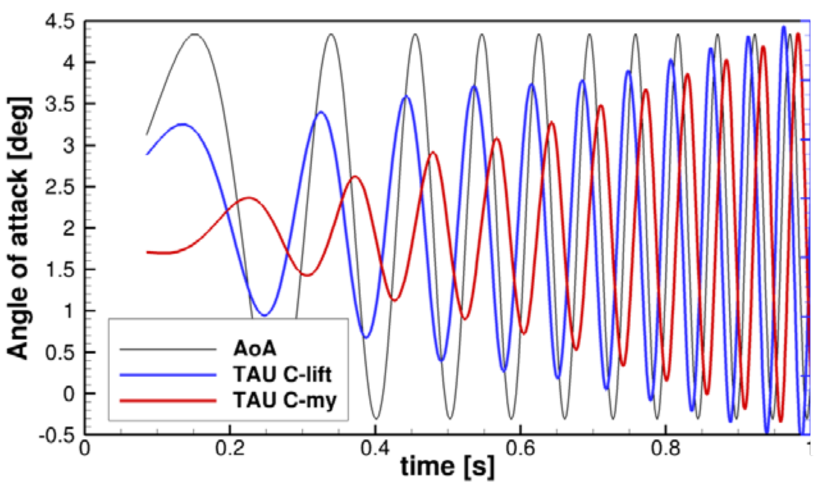
**Training maneuver**

Chirp pitching oscillation
up to reduced frequency **k=3**

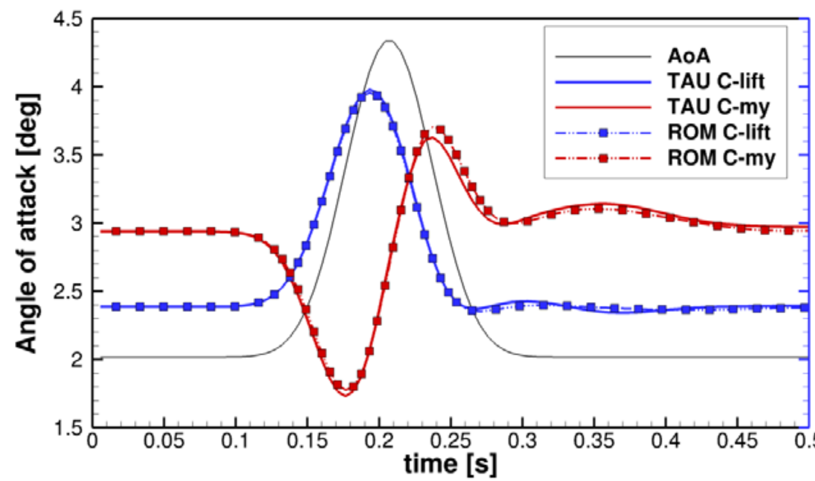
**Predicted maneuver**

(1-cos)–like pitching oscillation

**Training** @ Mach = 0.83, Re = 6.5*1e6

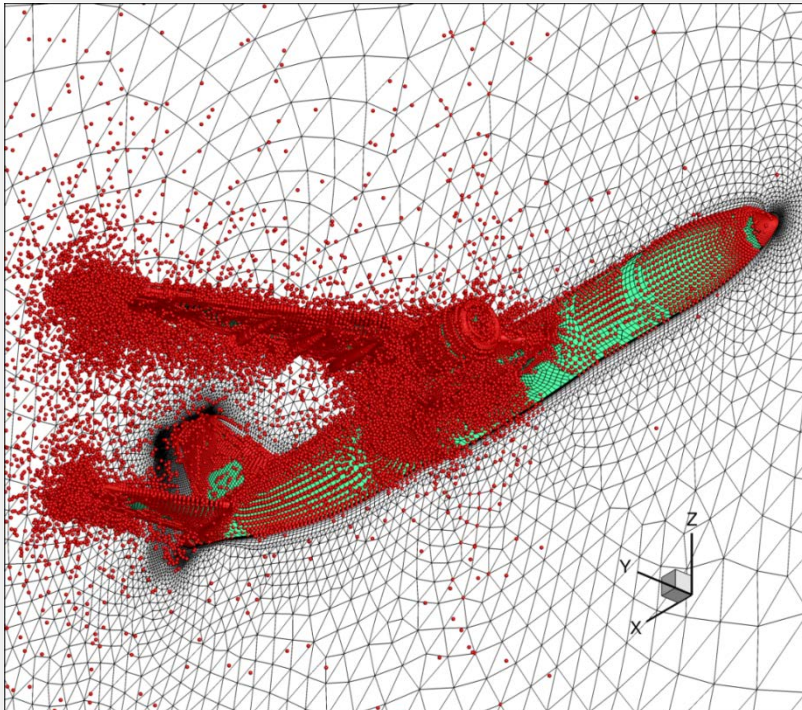


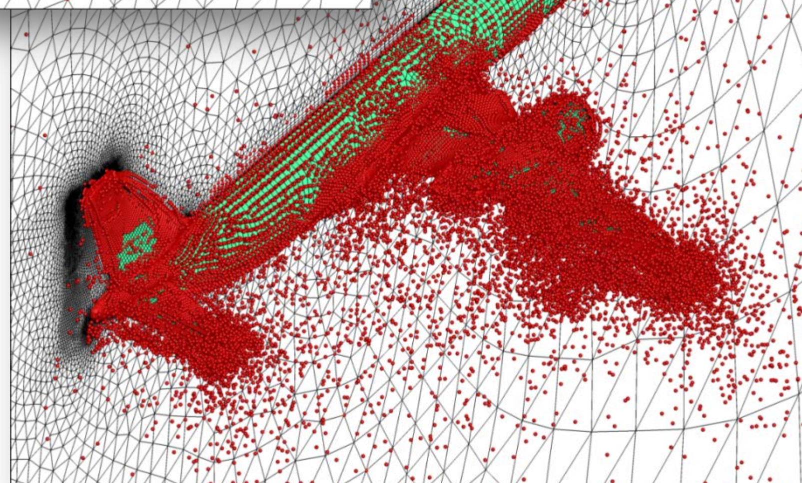

**ROM prediction** w/o MPE procedure

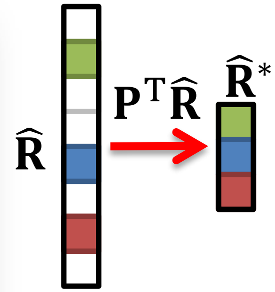



**ROM run-time** (48 cores): 1.3 h
**Speed-up**: 2.3

# Accelerated greedy MPE with rank-1 SVD update



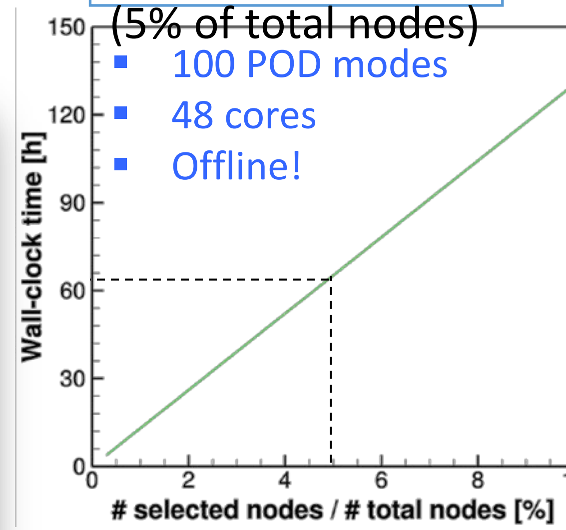






**Grid**
N° nodes: 3.8 Mi

**rank-1 SVD update**
**Greedy MPE**
N° nodes: 0.19 Mi
(5% of total nodes)

- 100 POD modes
- 48 cores
- Offline!



# Summary

The effectiveness of CFD-based **R**educed **O**rder **M**odels has been demonstrated for:

- o real-time applications
- o multi-disciplinary optimization (MDO)
  - static aeroelastic predictions
  - loads prediction and structural sizing
  - efficient selection of critical load cases
- o unsteady aerodynamics and maneuvers

# Outlook

- Include the greedy MPE selection in the ROM prediction for the full aircraft
- Investigate devide-and-conquer algorithm and coarse-grid residual evaluation as an alternative to greedy algorithm
- Apply the nonlinear unsteady least-squares ROM approach to discrete gusts
- What is the best training maneuver? → **ROM challenge …**
- Investigate alternatives to POD (DMD, isomap, …) for unsteady ROMs
- Consider interpolation-based approaches instead of residual minimization?!

**NEVER FORGET THE PHYSICS**

Nathan Kutz, Washington Uni

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- Part of the research leading to this work was supported by the **AEROGUST** project, funded by the European Commission under grant number 636053.
- COST Action TD1307: **EU-MORNET, Model Reduction Network.**

EU-MORNET
European Network for Model Reduction
cost

**German National Research Projects**

- Lufo-IV joint research project **AeroStruct**

**Internal DLR Projects**

- DLR multidisciplinary project **Digital-X**

DLR