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Mathematics for Business

Review Sheet

to be discussed on Tuesday/Wednesday 24/25 January

1. Consider the function $f(x) = \frac{x-1}{x+1}$.

   a) Determine the maximal domain of definition of f.

   b) Calculate $f'(x)$ and determine the regions where f is monotone increasing/decreasing.

   c) Determine the equation of the tangent to the graph of f at the point $(0, -1)$.

   d) Sketch the graph of f.

   e) Is f invertible? If so, determine f^{-1} along with its domain of definition.

2. Let $f(x) = x^2 e^x$.

   a) Determine the maximal domain of definition of f.

   b) Find the local extreme points and the inflection points of f.

   c) Sketch the graph of f.

   d) Calculate the area between the graph of f and the x–axis over the interval $[0, 2]$.

   e) Show that the equation $x^2 e^x = 8$ has exactly one solution $x \in [0, 2]$. Are there other solutions of the equation which do not lie in $[0, 2]$?

3. Evaluate the following integrals:

$$\int_0^1 x \ln(x+2) dx; \quad \int 3x^2 e^{x^3} dx.$$

4. Let $N(t)$ denote the number of people in a country whose homes have a telephone service. Suppose that the rate at which new people get access to a telephone service is proportional to the number of people who still have no access. Then $N(t)$ satisfies the differential equation

$$N'(t) = k(P - N(t)),$$

   where P is the population size and k is a positive constant.

   a) Find the solution of this equation if $N(0) = 0$.

   b) Determine the limit of $N(t)$ as $t \to \infty$.

5. For which values of a, b, c does the following system have solutions?

$$\begin{aligned} x_1 + 2x_2 + x_3 &= a \\ \frac{3}{4}x_1 + x_2 + \frac{7}{4}x_3 &= b \\ 3x_1 + 4x_2 + 7x_3 &= c. \end{aligned}$$

   Find the set of solutions if $a = 1, b = 1, c = 4$.

Please turn over!

6. For $a \in \mathbb{R}$ let

$$A = \begin{pmatrix} a & 1 & 4 \\ 2 & 1 & a^2 \\ 1 & 0 & -3 \end{pmatrix}.$$

a) Calculate A^2, $\det A$ and $\det(A^2)$.

b) Find all values of a for which A is invertible.

7. Solve the following linear programming problem with the help of the Simplex method:

$$\text{maximize } 10x_1 + \frac{25}{2}x_2 \qquad \text{subject to} \quad \begin{aligned} 5x_1 + \frac{25}{2}x_2 &\leq 100 \\ 9x_1 + \frac{15}{2}x_2 &\leq 90 \\ x_1 &\leq 7 \\ x_1, x_2 &\geq 0. \end{aligned}$$

8. Determine and classify the stationary points of

$$f(x, y) = y^3 + 3x^2y - 12x - 15y.$$

9. Assume that the solutions of the equation $e^{xy^2} - 2x - 4y + 3 = 0$ can be written in the form $y = h(x)$ when (x, y) is close to $(0, 1)$. Find $h'(0)$.

10. Examine the following problem with the help of Lagrange's method:

$$\text{maximize} \quad 20x + 39y - 2x^2 - 3y^2 \qquad \text{subject to } 4x + 6y = 24.$$

a) Calculate the only solution candidate (x^*, y^*, λ^*) to this problem.

b) Assuming that the point found in a) is a maximum, determine by approximately how much the optimal value increases if the constraint is changed to $4x + 6y = 24.5$.