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Mathematics for Business

Series 6

to be discussed on Wednesday, 18 January

1. Let $f(x,y) = 3xy^2 + 4x^3 - 3y^2 - 12x^2 + 1$. Find and classify the stationary points of f.

2. A firm produces two output goods A and B. The cost per day is

$$C(x,y) = \frac{2}{50}x^2 - \frac{1}{100}xy + \frac{1}{100}y^2 + 4x + 2y + 500,$$

when x units of A and y units of B are produced. The firm sells all it produces at prices 13 per unit of A and 8 per unit of B.
a) Write down the profit function $\pi(x,y)$.
b) Show that π is concave on $D = \{(x,y) \in \mathbb{R}^2 \,|\, x > 0, y > 0\}$.
c) Determine the daily production levels x and y which maximize profit.

3. Use Lagrange's method to solve the following problem:

$$\text{maximize } 10x^{\frac{1}{2}}y^{\frac{1}{3}} \quad \text{subject to } 2x + 4y = m.$$

a) Find the only solution candidate (x^*, y^*, λ^*) to this problem.
b) Assuming that the point found in a) is the maximum, write down the optimal value function $F(m)$ and calculate $F'(m)$ both directly and with the help of the Lagrange multiplier λ^*.

4. A household consumes x units of a good A, y units of B and z units of C. The utility function is given by

$$U(x,y,z) = 5x + 10y + 20z - \frac{1}{2}x^2 - \frac{1}{4}y^2 - z^2,$$

while the available income is 17. The prices for A, B and C are respectively given by 1, 2 and 4 (in a suitable unit).
a) Use Lagrange's method in order to determine how many units of A, B and C should be bought in order to maximize utility.
b) By approximately how much does the optimal utility increase if the available income increases by one unit?

5. Consider the problem

$$\text{maximize } 8x + 9y \quad \text{subject to } 4x^2 + 9y^2 \leq 100.$$

Determine all the solutions of the Kuhn–Tucker–conditions including the corresponding Lagrange multiplier λ.