Dr. Martin Stoll/Dr. Sara Grundel November 30, 2016
www.mpi-magdeburg.mpg.de/3166397/maths-for-business

Mathematics for Business
Series 5
to be discussed on Wednesday, 4 January

1. Use the Simplex method in order to solve the following linear programming problem:

$$\begin{aligned} \text{maximize} \quad & 3x_1 + 2x_2 + 3x_3 \\ \text{subject to} \quad & 2x_1 + x_2 + x_3 \leq 2 \\ & x_1 + 2x_2 + 3x_3 \leq 5 \\ & 2x_1 + 2x_2 + x_3 \leq 6 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

2. A firm produces two commodities A and B. The firm has three factories that jointly produce both commodities in the amounts per hour given in the following table:

	Factory 1	Factory 2	Factory 3
Commodity A	10	20	20
Commodity B	20	10	20

The firm receives an order for 300 units of A and 500 units of B. The costs per hour of running factories 1,2 and 3 are respectively 10000, 8000 and 11000.
a) Let $u_i, i = 1, 2, 3$, denote the number of hours for which factory i is used. Write down the linear programming problem of minimizing the costs of fulfilling the order.
b) Solve the problem found in a) by interpreting it as the dual of a maximization problem.

3. Consider
$$f(x, y, z) = e^{x^2(y+z)}; \quad g(x, y) = \frac{x - y}{x + y}.$$
Calculate the partial derivatives of first order of f and g as well as the Hessian of f.

4. Let $f(x, y) = xy$.
a) Draw the level curves $N_c = \{(x, y) \in \mathbb{R}^2 \mid f(x, y) = c\}$ for $c = 0, 1, 2$.
b) Calculate the vector $v = (\frac{\partial f}{\partial x}(x, y), \frac{\partial f}{\partial y}(x, y))$ and draw it at (x, y) for $(x, y) = (1, 1), (2, \frac{1}{2}) \in N_1$ respectively. What do you observe?
c) Let $g(x, y) = x+y$. Use your sketch of N_1 and the level curves of g in order to geometrically determine the minimum of $g(x, y)$ subject to the constraints $xy = 1, x > 0, y > 0$.

Please turn over!

5. Let $D = D(p, m)$ denote a typical consumer's demand for a particular commodity as a function of its price p and the consumer's own income m. The function

$$f(p, m) = \frac{pD(p, m)}{m}$$

represents the proportion of income spent on the commodity. Calculate $\frac{\partial f}{\partial m}(p, m)$ and use the result to show that f increases with m if $\mathrm{El}_m D(p, m) > 1$ and decreases if $\mathrm{El}_m D(p, m) < 1$.

6. Let $f(x, y, z) = z^4 e^{-xy^2}$ and define $h(s, t) = f(s + t^2, t - s, st)$. Calculate $\frac{\partial h}{\partial s}, \frac{\partial h}{\partial t}$ with the help of the chain rule. Check your answer by first calculating $h(s, t)$ and then differentiating.

7. Let $f(P, t)$ be the demand function for a commodity that depends on the price P before tax and the sales tax t per unit. Denoting by $g(P)$ the supply function, the equilibrium price is determined by the equation

$$f(P, t) = g(P). \tag{1}$$

Suppose that (1) defines P as a function of t, say $P = h(t)$. By writing (1) in the form $F(P, t) = 0$, calculate $h'(t)$ if $P = h(t)$. Use this formula in order to predict the effect of an increase in tax on the pre-tax price P under the (reasonable) assumptions that

$$g'(P) > 0, \ \frac{\partial f}{\partial P}(P, t) < 0, \ \frac{\partial f}{\partial t}(P, t) < 0.$$