Dr. Martin Stoll/Dr. Sara Grundel November 16, 2016

www.mpi-magdeburg.mpg.de/3166397/maths-for-business **Mathematics for**

Series 4

to be discussed on Wednesday, 30 November

1. (Open Leontief model) Consider an economy with three interlinked industries, each of which produces a single good. We denote by

$$\begin{aligned} a_{ij} &= \text{number of units of good i needed to produce one unit of good j,} \\ x_i &= \text{total number of units of good i produced in a certain year ,} \\ b_i &= \text{total number of units of good i demanded externally in a certain year.} \end{aligned}$$

   Let A, x, b be the 3×3 matrix and column vectors with entries a_{ij}, x_i, b_i respectively.

   a) Show that $x = Ax + b$.

   b) Given

$$A = \begin{pmatrix} \frac{1}{5} & 0 & \frac{1}{10} \\ \frac{2}{5} & \frac{1}{5} & \frac{1}{10} \\ 0 & \frac{2}{5} & \frac{3}{10} \end{pmatrix}, \quad b = \begin{pmatrix} 20 \\ 10 \\ 30 \end{pmatrix}$$

   determine x_1, x_2, x_3.

   c) Suppose that in the following year external demand for good 1 increases by 4 units. It is intended to meet this demand by increasing the production of good 1, at the same time reducing the number of units of good 2 in the production process of good 1. How many units of good 2 should be used to produce one unit of good 1 and how many units of goods 1 should be produced, when all other parameters remain unchanged?

2. Consider the following matrix A and the vector b which both depend on the parameter $a \in \mathbb{R}$:

$$A = \begin{pmatrix} 1 & 0 & -3 \\ 2 & 1 & a^2 \\ a & 1 & 4 \end{pmatrix}, \quad b = \begin{pmatrix} a \\ 2 \\ 2 \end{pmatrix}$$

   a) For which values of a is the matrix A invertible?

   b) For which values of a has the system $Ax = b$ (i) no, (ii) exactly one, (iii) infinitely many solutions? Determine the set of solutions in case (iii).

3. Let

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & -1 \end{pmatrix}.$$

   Find $A \cdot B$ and verify the formula $\det(A \cdot B) = \det A \cdot \det B$ by explicitly calculating the determinants on the left and on the right hand side.

Please turn over!

4. Calculate the following determinant by expansion in terms of the second row and the third column respectively:

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 0 & 11 \\ 2 & -1 & 0 & 3 \\ -2 & 0 & -1 & 3 \end{vmatrix}.$$

5, Examine whether the following matrices are positive or negative definite:

$$A = \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix}, \qquad A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}.$$