

Mathematics for Business**Series 3****to be discussed on Tuesday, 16 November**

1. a) Find the following integrals:

$$\int \frac{x^2}{2+x} dx; \quad \int_{-1}^2 x\sqrt{1+x} dx; \quad \int \frac{x^2}{\sqrt{1+x^3}} dx; \quad \int_0^{2\pi} \sin^2(x) dx.$$

b) Calculate the area that is enclosed by the graph of the function $y = f(x) = \sqrt{6-2x}$ and the two coordinate axes.

c) Consider the following demand and supply functions:

$$f_D(x) = \frac{6000}{x+50}, \quad f_S(x) = x+10.$$

Find the equilibrium price and compute the consumer and producer surplus.

2. Suppose that a firm's capital stock $K(t)$ satisfies the differential equation

$$K'(t) = I - \delta K(t),$$

where investment I is constant and $\delta K(t)$ denotes depreciation, with δ a positive constant.

a) Find the solution of the equation if the capital stock at time $t = 0$ is K_0 .

b) Let $\delta = 0.05$, $I = 10$ and examine the behaviour of the solutions when (i) $K_0 = 150$;

(ii) $K_0 = 250$.

3. The value of a new car depreciates continuously at the annual rate of 10%. How many years does it take for the car to lose 90% of its original value?

4. Suppose that the market value of a tree that is planted at time $t = 0$ is given by $P(t) = (t+5)^2$.

a) Assuming that interest is compounded continuously at a rate r , what is the present value of the tree?

b) At what time t^* should this tree be cut down in order to maximize its present value?

c) How does an increase/decrease in interest rate affect t^* ?

5. A construction firm wants to buy a building site and has the choice between three different payment schedules:

a) Pay €67000 in cash.

b) Pay €12000 per year for 8 years, where the first instalment is to be paid at once.

c) Pay €22000 in cash and thereafter €7000 per year for 12 years, where the first instalment is to be paid after one year.

By calculating the corresponding present values determine which schedule is least expensive if the interest rate is 11.5% and the firm has at least €67000 to spend in cash.