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Mathematics for Business

Series 1

(to be discussed on Wednesday, 2nd of November 2016)

1. Consider the equation
$$x^3 + 3x - 8 = 0.$$
   a) Show that this equation has a solution $x^* \in (1, 3)$.

   b) Show that the equation can only have one solution in $\mathbb{R}$ by examining the monotonicity of $f(x) = x^3 + 3x - 8$.

   c) Calculate an approximate value for x^* by carrying out two steps of Newton's method with initial value $x_0 = 1$.

2. a) At which points are the following functions differentiable? Calculate the derivatives where they exist.
$$f(x) = e^{x\ln(x)}; \quad h(t) = \sqrt{\cos(t)}, \quad g(u) = \frac{1}{(u^2+u+1)^5}.$$
   b) Suppose that f is differentiable at x. Find an expression for $g'(x)$, when $g(x) = f(x)^2$.

3. The function $\tan : (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$ has an inverse function $\arctan : \mathbb{R} \to \mathbb{R}$, given by
$$\arctan(y) = x \Longleftrightarrow y = \tan(x).$$
   Sketch the graph of arctan and show that $\arctan'(y) = \dfrac{1}{1+y^2}, y \in \mathbb{R}$.

4. a) Assume that $D(p) = 18 - 2p, 0 \leq p \leq 9$ is the demand function of a particular commodity. At which price $\bar{p}$ does a 1% price reduction lead to a 2% increase in demand?

   b) Let $D(p)$ be the demand function of a product and $R(p) = pD(p)$ the corresponding revenue. Express $\mathrm{El}_p R(p)$ in terms of $\mathrm{El}_p D(p)$.

5. For each of the following functions find the domain of definition, determine the local extrema (i.e. local maxima and minima) as well as the inflection points and sketch the graph.

   a) $f(x) = \dfrac{x}{1+x^2}$;

   b) $g(x) = \ln(1+x) - x + \frac{1}{2}x^2 - \frac{1}{6}x^3$.

   Determine the global extrema of f on $[-2, 2]$ and of g on $[-\frac{1}{2}, 2]$.