Otto-Von-Guericke-University-Magdeburg
Department of Mathematics-Institute for Analysis and Numerik
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Computational Methods for Systems and Control Theory

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Model Reduction of Dynamical Systems - 5

Deadline for homework: 23/06/2015

Task: 1 (Properties of the matrix sign function)

Assume $Z \in \mathbb{C}^{n\times n}$ with no eigenvalues on the imaginary axis. Show that it holds:

(a) $\text{sign}(Z)^2 = I_n$, i.e., $\text{sign}(Z)$ is a square root of the identity matrix;

(b) $\text{sign}(T^{-1}ZT) = T^{-1}\text{sign}(Z)T$ for all nonsingular $T \in \mathbb{C}^{n\times n}$

(c) If Z is stable, then

$$\text{sign}(Z) = -I_n, \qquad \text{sign}(-Z) = I_n$$

(d) Define $Z_0 \leftarrow Z$, $Z_{k+1} \leftarrow \frac{1}{2}(Z_k + Z_k^{-1})$, $k = 0, 1, 2, \ldots$
Show that the above scheme converges to $\text{sign}(Z)$

Task: 2 (Solving Sylvester equations via the matrix sign function)

a) Consider the Sylvester equation

$$AX + XB + C = 0, \tag{1}$$

with $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times m}$ and $C \in \mathbb{R}^{n\times m}$. Assume that A and B are asymptotically stable matrices and that X is a solution of eq. (1). Show that it holds

$$\text{sign}\left(\begin{bmatrix} A & C \\ 0 & -B \end{bmatrix}\right) = \begin{bmatrix} -I & 2X \\ 0 & I \end{bmatrix}.$$

b) Show that instead of iterating on $\begin{bmatrix} A & C \\ 0 & -B \end{bmatrix}$, one can compute X via an iteration on A, B, C.

Task: 3 (The extended Krylov subspace method for the Lyapunov equation)

As a fast and reliable method for computing low-rank approximations to solutions of Lyapunov equations, one can use the so-called extended Krylov subspace method (EKSM). The main idea is to construct V as an orthogonal basis of the union of the Krylov spaces $\mathcal{K}_q(A, b)$ and $\mathcal{K}_q(A^{-1}, A^{-1}b)$. Implement an iterative method applicable to (A, b) which in each step checks the accuracy in terms of the relative residual $||AY_{2q} + Y_{2q}A^T + bb^T||_F/||bb^T||_F$. Try your method for the CD player model for different values of q. Compare your results with the method from theoretical exercise 13 with shifts parameters uniformly chosen in the interval $[0, 1000]$.

Send your routines to *imahmad@mpi-magdeburg.mpg.de*. The filename should include your name and the corresponding exercise sheet number as well as the exercise number, e.g., name-hw1e5. In case of several files please hand in a compressed file. Moreover, please print the source code of your routine and hand it in together with the other exercises.