Otto-Von-Guericke-University-Magdeburg | Summer Term 2015
Department of Mathematics-Institute for Analysis and Numerik | Prof. Peter Benner
Max-Planck-Institute-Magdeburg | Dr. Lihong Feng
Computational Methods for Systems and Control Theory | Dr. M. Ilyas Ahmad

Model Reduction of Dynamical Systems - 4

Task: 1 (Asymptotic waveform evaluation)

(a) Use the asymptotic waveform evaluation method and derive an expression for the numerator and denominator coefficients of a Padé approximation $H_r(s) = \sum_{i=0}^{\infty} \hat{m}_i s^i$ corresponding to a linear time invariant system $H(s) = \sum_{i=0}^{\infty} m_i s^i$, where $\hat{m}_i = m_i$, for $i = 1, \ldots, r$ which represent the moments at $s = 0$. Also identify expressions for the associated poles and residues of $H_r(s)$.

(b) Implement the asymptotic waveform evaluation method using the pole-residue framework. Test your program by means of the model of a CD player which you find as *CDPlayer.mat* on the course homepage. Evaluate the transfer function

$$H(i\omega) = C(i\omega I - A)^{-1}B + D$$

for original and reduced-order model over the frequency interval $\omega \in [10^{-1}, 10^5]$. Use 10 000 logarithmically distributed sample points. Plot the *gain* of the transfer function, i.e. $20 \cdot \log_{10} |(H(j\omega)|$ on a logarithmic x-scale by using the MATLAB command *semilogx*(ω, H_ω). Use four different values of r, i.e., $r = 5$, $r = 8$, $r = 10$ and $r = 12$ and interpret the results.

Task: 2 (Model reduction by interpolation)

Given a SISO LTI system

$$\dot{x}(t) = Ax(t) + bu(t),$$
$$y(t) = c^T x(t).$$

Assume that a reduced-order model is given by a Petrov-Galerkin type projection $\mathcal{P} = VW^T$, i.e.

$$\hat{A} = W^T AV,\ \hat{b} = W^T b,\ \hat{c} = V^T c.$$

Show that if $(\sigma I - A)^{-1} b \in \text{range}(V)$ and $(\sigma I - A^T)^{-1} c \in \text{range}(W)$, for $\sigma \in \mathbb{C} \backslash \{\Lambda(A) \cup \Lambda(\hat{A})\}$, the reduced-order transfer function $\hat{H}(s)$ is a Hermite interpolant of $H(s)$ in σ, i.e., it holds

$$H(\sigma) = \hat{H}(\sigma), \quad H'(\sigma) = \hat{H}'(\sigma).$$