Otto-Von-Guericke-University-Magdeburg
Department of Mathematics-Institute for Analysis and Numerik
Max-Planck-Institute-Magdeburg
Computational Methods for Systems and Control Theory

Summer Term 2015
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Model Reduction of Dynamical Systems - 3

Deadline for homework: 26/05/2015

Task: 1 (Reachability Grammian and Reachability Subspace)

Show that the finite reachability gramians $P(t) = \int_0^t e^{A\tau} B B^T e^{A^T \tau} d\tau$, $0 < t < \infty$ have the following properties:

- $P(t) = P^T(t) \geq 0$,
- $\text{range}(P(t)) = \text{range}(R(A,B))$, where $R(A,B) = [B,\ AB,\ A^2B \ldots A^{n-1}B \ldots]$
  (The columns of the reachability grammian span the reachability subspace)

Task: 2 (Model reduction by balanced truncation)

Implement the method of balanced truncation introduced in the course. If you do not have access to the control system toolbox, you can use the routine *lyap_sgn_fac.m* from the course homepage to compute approximations to the Cholesky factors of the solutions to the Lyapunov equations. Try your program by means of the model of a beam which you find as *beam.mat* on the course homepage. Evaluate the transfer function

$$H(i\omega) = C(i\omega I - A)^{-1} B$$

for original and reduced-order model over the frequency interval $\omega \in [10^{-2}, 10^4]$. Use 10 000 logarithmically distributed sample points. Plot the *gain* of the transfer function, i.e. $20 \cdot \log_{10} |(H(j\omega)|$ on a logarithmic x-scale by using the MATLAB command *semilogx*(ω, H_ω).

Send your routines to *imahmad@mpi-magdeburg.mpg.de*. The filename should include your name and the corresponding exercise sheet number as well as the exercise number, e.g., name-hw1e5. In case of several files please hand in a compressed file. Moreover, please print the source code of your routine and hand it in together with the other exercises.