Otto-Von-Guericke-University-Magdeburg
Department of Mathematics-Institute for Analysis and Numerik
Max-Planck-Institute-Magdeburg
Computational Methods for Systems and Control Theory

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Prof. Peter Benner
Dr. Lihong Feng
Dr. M. Ilyas Ahmad

Model Reduction of Dynamical Systems - 3

Exercise: 3.1 (Balancing-free square root (BFSR) method)

We have already discussed the square-root balanced truncation technique for model reduction in Excercise 2.1. Another way of model reduction is to use the balancing-free square root (BFSR) algorithm. Analogue to the standard square-root balanced truncation approach, one has to compute the Cholesky factors S and R of the solutions of the Lyapunov equations and the corresponding SVD of those factors, i.e.,

$$AP + PA^T + BB^T = 0, \quad A^TQ + QA + C^TC = 0, \quad SR^T = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}.$$

The left and right projection matrices for the computation of a reduced-order model of dimension r now are given as $T_l = (Q_1^T P_1)^{-1} Q_1^T$ and $T_r = P_1$, with

$$S^T U_1 = \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} \hat{R} \\ 0 \end{bmatrix}, \quad R^T V_1 = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} \bar{R} \\ 0 \end{bmatrix},$$

and $P_1, Q_1 \in \mathbb{R}^{n \times r}$ have orthonormal columns and $\hat{R}, \bar{R} \in \mathbb{R}^{r \times r}$ are upper triangular.

Show that the reduced-order system is equivalent to a balanced system and that it satisfies the same error bound as the one obtained by the standard square root BT method.

Exercise: 3.2 (Balanced model reduction for non-minimal systems)

Consider a system which is neither controllable nor observable, i.e.,

$$\mathcal{K} = \operatorname{rank}\left(\begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix}\right) = k_1 < n \quad \text{and} \quad \mathcal{O} = \operatorname{rank}\left(\begin{bmatrix} C \\ CA^T \\ \vdots \\ C(A^T)^{n-1} \end{bmatrix}\right) = k_2 < n.$$

Show that if $Y \in \mathbb{R}^{n \times k_1}$ and $Z \in \mathbb{R}^{n \times k_2}$ are low rank factors that satisfy $P = YY^T$ and $Q = ZZ^T$, a balanced reduced-order model can be obtained by projection matrices $T_l = \Sigma_1^{-\frac{1}{2}} V_1 Z^T$ and $T_r = YU_1\Sigma_1^{-\frac{1}{2}}$, where $Y^TZ = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$.