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Computational Methods for Systems and Control Theory

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Model Reduction of Dynamical Systems - 2

Exercise: 1 (Balanced realizations)

Given a minimal LTI system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0,$$
$$y(t) = Cx(t) + Du(t).$$

Show that a balanced realization is given by the state-space transformation

$$T := \Sigma^{-\frac{1}{2}} V^T R,$$

where $P = S^T S$ and $Q = R^T R$ (e.g., Cholesky decompositions) satisfy the pair of Lyapunov equations

$$AP + PA^T + BB^T = 0,$$
$$A^T Q + QA + C^T C = 0$$

and

$$SR^T = U\Sigma V^T$$

is the SVD of SR^T.

Hint: First note that $T^{-1} = S^T U \Sigma^{-\frac{1}{2}}$, then the result follows by simple algebraic manipulations.

Exercise 2 (Output Error Bound)

Given an LTI system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0,$$
$$y(t) = Cx(t) + Du(t).$$

with the input output relation in frequency domain, $y(s) = (C(sI - A)^{-1}B + D)u(s) := G(s)u(s)$. Show that the error bound for a reduced model $\hat{y}(s) = (\hat{C}(sI - \hat{A})^{-1}\hat{B} + \hat{D})u(s) := \hat{G}(s)u(s)$, is

$$\|y - \hat{y}\|_\infty \leq \|G(s) - \hat{G}(s)\|_{\mathcal{H}_2} \|u\|_{\mathcal{L}_2},$$

where the ∞-norm is the vector norm in the Euclidean space for any fixed t. .