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Computational Methods for Systems and Control Theory

Summer Term 2015
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Model Reduction of Dynamical Systems - 1

Excercise: 1 (Properties of Singular Value Decomposition (SVD))

Let $r = \text{rank}(A)$, $A = U\Sigma V^T$ be the singular value decomposition with orthonormal matrices $U = \begin{bmatrix} u_1 & \cdots & u_m \end{bmatrix} \in \mathbb{R}^{m\times m}$ and $V = \begin{bmatrix} v_1 & \cdots & v_n \end{bmatrix} \in \mathbb{R}^{n\times n}$. Show the following statements:

a) Schmidt-Eckart-Young-Mirsky-Theorem: If $k < r$ and $A_k = \sum_{i=1}^{k} \sigma_i u_i v_i^T$, then it holds:

$$\min_{\text{rank}(B)=k} \|A - B\|_2 = \|A - A_k\|_2 = \sigma_{k+1},$$

that means A_k is the best rank-k approximation of A.

Hint: Realize that $\text{rank}(A_k) = k$. Then select a matrix $B \in \mathbb{R}^{m\times m}$ with $\text{rank}(B) = k$. There exist orthonormal vectors $x_1, \ldots, x_{n-k}$ such that $ker(B) = \text{span}\{x_1, \ldots, x_{n-k}\}$ and it holds: $\text{span}\{x_1, \ldots, x_{n-k}\} \cap \text{span}\{v_1, \ldots, v_{k+1}\} \neq \{\emptyset\}$. Let z be a vector of this intersection with $\|z\| = 1$. Then it holds $Bz = 0$. Making use of the representation of Az one can show the necessary estimate.

b) It holds:

$$\ker(A) = \{v_{r+1}, \ldots, v_n\},$$
$$\text{range}(A) = \{u_1, \ldots, u_r\}.$$

c) It holds:

$$\|A\|_F = \sqrt{\sigma_1^2 + \cdots + \sigma_r^2}$$

Excercise: 2 (Laplace Transform)

a) Find Laplace transform of the following functions:

(i) $f(t) = 2t^2 - 3t + 5$.
(ii) $f(t) = t^2 e^{-2t}$.
(iii) $f(t) = sin(2t)cos(2t)$.
(iv) $f(t) = sin(2t) + e^{-3t}cos(2t)$.

b) Invert each of the following Laplace transforms:

(i) $F(s) = \frac{4}{s^5}$.
(ii) $F(s) = \frac{32}{s(s^2+16)}$.

c) Using Laplace transform, solve the following ordinary differential equations:

(i) $\ddot{x}(t) + x(t) = t, \quad x(0) = 0, \ \dot{x}(0) = 2$.
(ii) $\ddot{x}(t) + 2\dot{x}(t) + 5x = 8e^{-3t}, \quad x(0) = \dot{x}(0) = 0$.