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Computational Methods for Systems and Control Theory

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Model Reduction of Dynamical Systems - 3

Deadline for homework: 20/05/2014

Task: 1 (Asymptotic waveform evaluation)

Implement the asymptotic waveform evaluation method to identify a Padé approximation $H_r(s)$ of a linear time invariant system $H(s)$. Try your program by means of the model of a CD player which you find as *CDPlayer.mat* on the course homepage. Evaluate the transfer function

$$H(i\omega) = C(i\omega I - A)^{-1}B + D$$

for original and reduced-order model over the frequency interval $\omega \in [10^{-1}, 10^5]$. Use 10 000 logarithmically distributed sample points. Plot the *gain* of the transfer function, i.e. $20 \cdot \log_{10} |(H(j\omega)|$ on a logarithmic x-scale by using the MATLAB command *semilogx*(ω, H_ω). Use four different values of r, i.e., $r = 5$, $r = 8$, $r = 10$ and $r = 12$ and interpret the results.

Task: 2 (Model order reduction by the Arnoldi iteration)

Write a MATLAB routine which reads in an LTI realization (A, B, C, D), computes an orthogonal basis $V = [v_1, \ldots, v_k]$ for the Krylov subspace $\mathcal{K}_k(A, B)$ by means of the Arnoldi iteration discussed in the course and constructs a reduced-order realization as

$$\hat{A} = V^T AV,\ \hat{B} = V^T B,\ \hat{C} = CV,\ \hat{D} = D.$$

Again use *CDPlayer.mat* and evaluate the transfer function

$$H(i\omega) = C(i\omega I - A)^{-1}B + D$$

for original and reduced-order model similar to the first task. Test several values of k and interpret the results.

Task: 3 (Model reduction by interpolation)

Given a SISO LTI system

$$\dot{x}(t) = Ax(t) + bu(t),$$
$$y(t) = c^T x(t).$$

Assume that a reduced-order model is given by a Petrov-Galerkin type projection $\mathcal{P} = VW^T$, i.e.

$$\hat{A} = W^T AV,\ \hat{b} = W^T b,\ \hat{c} = V^T c.$$

Show that if $(\sigma I - A)^{-1}b \in \text{range}(V)$ and $(\sigma I - A^T)^{-1}c \in \text{range}(W)$, for $\sigma \in \mathbb{C}\backslash\{\Lambda(A) \cup \Lambda(\hat{A})\}$, the reduced-order transfer function $\hat{H}(s)$ is a Hermite interpolant of $H(s)$ in σ, i.e., it holds

$$H(\sigma) = \hat{H}(\sigma), \quad H'(\sigma) = \hat{H}'(\sigma).$$

Task: 4 (The iterative rational Krylov algorithm)

Implement the iterative rational Krylov algorithm (IRKA) discussed in the course for a SISO linear time invariant system. Use the model of the beam available on the course homepage as *beam.mat*. Make the initial selection of the interpolation points random and closed under conjugation. Evaluate the transfer function for the original $H(i\omega)$ and the reduced-order hermite interpolant $H_r(i\omega)$, similar to Task 1 and 2.

Send your routines to *imahmad@mpi-magdeburg.mpg.de*. The filename should include your name and the corresponding exercise sheet number as well as the exercise number, e.g., name-hw1e5. In case of several files please hand in a compressed file. Moreover, please print the source code of your routine and hand it in together with the other exercises.