On Stability, Passivity and Reciprocity Preservation of ESVD MOR

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Abstract The reduction of parasitic linear subcircuits is one of many issues in model order reduction (MOR) for VLSI design. This issue is well explored, but recently the incorporation of subcircuits from different modelling sources into the circuit model has led to new structural aspects: so far, the number of elements in the subcircuits was significantly larger than the number of connections to the whole circuit, the so called pins or terminals. This assumption is no longer valid in all cases such that the simulation of these circuits or rather the reduction of the model requires new methods. In [6, 15, 17], the extended singular value decomposition based model order reduction (ESVD MOR) algorithm is introduced as a way to handle this kind of circuits with a massive number of terminals. Unfortunately, the ESVD MOR approach has some drawbacks because it uses the SVD for matrix factorizations. In [5, 22] the truncated SVD (TSVD) as an alternative to the SVD within the ESVD MOR is introduced. In this paper we show that ESVD MOR as well as the modified approach is stability, passivity, and reciprocity preserving under reasonable assumptions.

1 Introduction

MOR has become an important tool in the preprocessing of circuit simulation over the last decades. The original model which results from the mathematical modeling with the help of, e. g., modified nodal analysis [11] has to be simplified due to its complexity. One special part of this simplification for VLSI design is the MOR of parasitic linear interconnect circuits [7]. These circuits appear in form of substructures in the design of integrated circuits (ICs). They contain linear elements which do not necessarily have large influence on the result of the simulation.

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There are applications in which the structure of these parasitic linear subcircuits has been changing recently in the following sense. So far, the number of elements in these interconnect circuits was significantly larger than the number of connections to the whole circuit, the so-called pins or terminals. This assumption is not true anymore in many cases. Power grid networks which supply elements of large circuits with energy are of this special form [23, 26]. Often these power grids are realized as an extra layer of elements in between the layers of transistors. The fact that they need connections to the elements for power supply explains the high number of pins. The same problem appears in connection with synchronization. In clock distribution networks, the clock signal is distributed from a common point to all the elements that need it for synchronization [14]. For simulating these special subcircuits, new methods are needed. In some applications, a lot of terminal signals behave similarly so that it is possible to compress the input-/output matrices of the transfer function in such a way that the I/O behavior can be realized through a few so-called virtual inputs/outputs. As a consequence we deal with these virtual terminals, the number of which is much less than the original number of terminals. This allows the use of well known MOR methods like balanced truncation or Krylov subspace methods to reduce the number of inner nodes. The basic work was done a few years ago with the introduction of the (E)SVD MOR approach [6, 15–17, 25]. In this paper we review the existing ESVD MOR approach [6, 17] and discuss stability, passivity, and reciprocity of the computed reduced-order model. As most linear subcircuits with a massive number of pins represent RC(L) circuits and contain no active devices, they are modelled as passive and thus stable systems. Therefore, it is generally important to compute reduced-order models that share these properties with the original model. Moreover, preserving reciprocity allows the synthesis of the reduced-order model as circuit [20].

In the following section, we review the fundamentals of the ESVD MOR approach. We introduce the moments of a transfer function of the circuit describing system and show how to use the information in these moments in order to reduce the number of terminals. In this way we achieve a very compact model. The preservation of stability and passivity in the reduced-order models is the topic of Section 3. We introduce basic definitions and prove that the ESVD MOR approach is stability, passivity, and reciprocity preserving under certain conditions. A numerical example shows the passivity preservation. In Section 4, we sum up the results and outline future research activities.

2 The Extended SVD MOR Approach

We consider a linear time-invariant continuous-time descriptor system, which can be represented as

$$\begin{aligned} C\dot{x}(t) &= -Gx(t) + Bu(t), \quad x(0) = x_0, \\ y(t) &= Lx(t), \end{aligned} \tag{1}$$

where $C, G \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m_{in}}$, $L \in \mathbb{R}^{m_{out} \times n}$, $x(t) \in \mathbb{R}^n$ contains internal state or descriptor variables, $u(t) \in \mathbb{R}^{m_{in}}$ is the vector of input variables, e.g., containing the terminal currents, $y(t) \in \mathbb{R}^{m_{out}}$ is the output vector, e.g., containing the terminal voltages, $x_0 \in \mathbb{R}^n$ is the initial value, and n is the number of state/descriptor variables, called the *order* of the system. The original linear system (1) to be reduced has the following transfer function in frequency domain:

$$H(s) = L(sC + G)^{-1}B. \quad (2)$$

The number of inputs m_{in} and the number of outputs m_{out} are not necessarily equal. Further on we can define the *i-th block moment of (2)* as

$$\mathbf{m}_i = L(-G^{-1}C)^i G^{-1}B, \quad i = 0, 1, \dots \quad (3)$$

in terms of $\mathbf{m}_i$ as an $m_{out} \times m_{in}$ matrix

$$\mathbf{m}_i = \begin{bmatrix} m_{1,1}^i & m_{1,2}^i & \dots & m_{1,m_{in}}^i \\ m_{2,1}^i & m_{2,2}^i & \dots & m_{2,m_{in}}^i \\ \vdots & \vdots & \ddots & \vdots \\ m_{m_{out},1}^i & m_{m_{out},2}^i & \dots & m_{m_{out},m_{in}}^i \end{bmatrix}. \quad (4)$$

Note that the moments in (3) are equal to the coefficients of the Taylor series expansion of (2) about $s = 0$. The expansion about $s = s_0 \neq 0$ leads to *frequency shifted moments* defined as

$$\mathbf{m}_i(s_0) = L(-(s_0C + G)^{-1}C)^i (s_0C + G)^{-1}B, \quad i = 0, 1, \dots \quad (5)$$

In the ESVD MOR method [6, 17], the information of a combination of these moments to create a decomposition of (2) is used in the following way. For being able to allow terminal reduction for inputs and outputs separately, w.l.o.g. we use r different block moments forming two moment matrices, the input response matrix M_I and the output response matrix M_O , as follows:

$$M_I = \begin{bmatrix} \mathbf{m}_0 \\ \mathbf{m}_1 \\ \vdots \\ \mathbf{m}_{r-1} \end{bmatrix}, \quad M_O = \begin{bmatrix} \mathbf{m}_0^T \\ \mathbf{m}_1^T \\ \vdots \\ \mathbf{m}_{r-1}^T \end{bmatrix}. \quad (6)$$

Note that it is also possible to use different numbers of block moments to create M_I and M_O and that column k of M_I represents the coefficients (moments) of the series expansion of (2) at all outputs corresponding to input k . Similarly, each column k of M_O represents the coefficients of output k corresponding to all inputs. We assume the number of rows in each matrix to be larger than the number of columns. If not, r has to be increased.

Applying the SVD to these matrices, we can obtain a low rank approximation

$$M_I = U_I \Sigma_I V_I^T \approx U_{I_{r_i}} \Sigma_{I_{r_i}} V_{I_{r_i}}^T, \quad M_O = U_O \Sigma_O V_O^T \approx U_{O_{r_o}} \Sigma_{O_{r_o}} V_{O_{r_o}}^T, \quad (7)$$

where

- $U_I = \begin{bmatrix} U_{I_{r_i}}, U_{I_{(rm_{out}-r_i)}} \end{bmatrix}$ is an $r \cdot m_{out} \times r \cdot m_{out}$ orthogonal matrix,
- $V_I = \begin{bmatrix} V_{I_{r_i}}, V_{I_{(m_{in}-r_i)}} \end{bmatrix}$ is an $m_{in} \times m_{in}$ orthogonal matrix,
- $U_O = \begin{bmatrix} U_{O_{r_o}}, U_{O_{(rm_{in}-r_o)}} \end{bmatrix}$ is an $r \cdot m_{in} \times r \cdot m_{in}$ orthogonal matrix,
- $V_O = \begin{bmatrix} V_{O_{r_o}}, V_{O_{(m_{out}-r_o)}} \end{bmatrix}$ is an $m_{out} \times m_{out}$ orthogonal matrix,
- Σ_I and Σ_O are $r \cdot m_{out} \times m_{in}$ and $r \cdot m_{in} \times m_{out}$ diagonal matrices,

whereas

- $\Sigma_{I_{r_i}}$ and $\Sigma_{O_{r_o}}$ are $r_i \times r_i$ and $r_o \times r_o$ diagonal matrices,
- $V_{I_{r_i}}$ and $V_{O_{r_o}}$ are $m_{in} \times r_i$ and $m_{out} \times r_o$ isometric matrices that contain the dominant column subspaces of M_I and M_O ,
- $U_{I_{r_i}}$ and $U_{O_{r_o}}$ are $rm_{out} \times r_i$ and $rm_{in} \times r_o$ isometric matrices that are not used any further.

$r_i \leq m_{in}$ and $r_o \leq m_{out}$ are the numbers of significant singular values as well as the numbers of the reduced virtual input and output terminals. Due to the fact that the important information about the dependencies of the I/O-ports is hidden in the matrices $V_{I_{r_i}}^T$ and $V_{O_{r_o}}^T$, approximations of B and L using the factors in (7) lead to

$$B \approx B_r V_{I_{r_i}}^T \text{ and } L \approx V_{O_{r_o}} L_r. \quad (8)$$

Here, $B_r \in \mathbb{R}^{n \times r_i}$ and $L_r \in \mathbb{R}^{r_o \times n}$ are computed by applying the Moore-Penrose pseudoinverse [19] (denoted by $(\cdot)^+$) of $V_{I_{r_i}}^T$ and of $V_{O_{r_o}}$ to B and L , respectively. In detail, this means

$$B_r = B V_{I_{r_i}} (V_{I_{r_i}}^T V_{I_{r_i}})^{-1} = B V_{I_{r_i}}^{T+} = B V_{I_{r_i}} \quad (9)$$

and

$$L_r = (V_{O_{r_o}}^T V_{O_{r_o}})^{-1} V_{O_{r_o}}^T L = V_{O_{r_o}}^+ L = V_{O_{r_o}}^T L, \quad (10)$$

since $V_{I_{r_i}}$ and $V_{O_{r_o}}$ are isometric. As a consequence we get a new internal transfer function $H_r(s)$ by using the approximation

$$H(s) \approx \hat{H}(s) = V_{O_{r_o}} \underbrace{L_r (G + sC)^{-1} B_r V_{I_{r_i}}^T}_{:=H_r(s)}. \quad (11)$$

This terminal reduced transfer function $H_r(s)$ can be further reduced to

$$\tilde{H}_r(s) = \tilde{L}_r (\tilde{G} + s\tilde{C})^{-1} \tilde{B}_r \approx H_r(s) = L_r (G + sC)^{-1} B_r \quad (12)$$

by some established MOR method, e.g., balanced truncation or a Krylov subspace method, see [2, 4, 21] for introductions to linear model reduction techniques. We end up with a very compact terminal reduced and reduced-order model $\tilde{H}_r(s)$, i. e.

$$H(s) \approx \hat{H}(s) \approx \hat{H}_r(s) = V_{O_{ro}} \tilde{H}_r(s) V_{I_{ri}}^T. \quad (13)$$

Note that the well-known SVD MOR approach [6] can be considered as a special case of ESVD MOR, using only one moment and one SVD, e. g. $r = 1$, and using $\mathbf{m}_0$ as moment.

The ESVD MOR approach as described above is not appropriate for really large-scale circuit systems as it employs the full SVD of the moment matrices. Observing that the computation of the reduced-order model only needs the leading block-columns $U_{I_{ri}}, V_{I_{ri}}, U_{O_{ro}}, V_{O_{ro}}$ of the orthogonal matrices computed within the SVD, the expensive SVD can be replaced by a truncated SVD which can be computed cheaply employing sparsity of the involved matrices using Krylov subspace or Jacobi-Davidson methods [12, 13]. Efficient algorithms based on the first approach are suggested in [5, 22] while the Jacobi-Davidson version is under current investigation.

3 Stability, Passivity, and Reciprocity

In this section we establish some facts on the preservation of stability, passivity, and reciprocity in ESVD MOR reduced-order models.

3.1 Stability

In order to discuss stability of descriptor systems we need the following definition and lemma which can be found, e.g., in [4].

Definition 1. The descriptor system (1) is called *asymptotically stable* if $\lim_{t \rightarrow \infty} x(t) = 0$ for all solutions $x(t)$ of $C\dot{x}(t) = -Gx(t)$.

Lemma 1. Consider a descriptor system (1) with a regular matrix pencil $\lambda C + G$. The following statements are equivalent:

1. System (1) is asymptotically stable.
2. All finite eigenvalues of the pencil $\lambda C + G$ lie in the open left half-plane.

Using the results from Lemma 1 we are able to formulate the following theorem:

Theorem 1. Consider an asymptotically stable system (1) with its transfer function (2). The ESVD MOR reduced-order system corresponding to (13) is asymptotically stable iff the inner reduction (12) is stability preserving.

Proof. It is obvious that none of the approximations (8), (11) and (13) change the eigenvalues of $\lambda C + G$. With Lemma 1 and the assumption that (12) is stability preserving it directly follows that the ESVD MOR approach is stability preserving. $\square$

A possible stability preserving model reduction method that can be applied along the lines of Theorem 1 is balanced truncation for regular descriptor systems, see [3, 18, 24].

3.2 Passivity

Showing that the ESVDMOR approach is passivity preserving turns out to be more difficult. First we note that a system is *passive* iff its transfer function is positive real [1]. The following definition of positive realness can be found, e.g., in [8].

Definition 2. The transfer function (2) is positive real iff the following three assumptions hold:

1. $H(s)$ has no poles in $\mathbb{C}_+ = \{s \in \mathbb{C} \mid \operatorname{Re} s > 0\}$, i. e. the system is stable if additionally there are no multiple poles on $i\mathbb{R}$,
2. $H(\bar{s}) = \overline{H(s)}$ for all $s \in \mathbb{C}$,
3. $\operatorname{Re}(x^H H(s)x) \geq 0$ for all $s \in \mathbb{C}_+$ and $x \in \mathbb{C}^m$.

For passive systems we have to assume that the number of inputs is equal to the number of outputs: $m_{in} = m_{out} = m$. Due to the fact that we often deal just with parasitic linear RLC circuits we furthermore assume $L = B^T$ such that

$$H(s) = B^T (sC + G)^{-1} B. \quad (14)$$

As a result of Modified Nodal Analysis (MNA) modeling the system has a well defined block structure [8] and we get

$$\begin{aligned} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \dot{x} + \begin{bmatrix} G_1 & G_2 \\ -G_2^T & 0 \end{bmatrix} x &= \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u, \\ y &= \begin{bmatrix} B_1 & 0 \end{bmatrix} x, \end{aligned} \quad (15)$$

where G_1, C_1, C_2 are symmetric, $G_1, C_1 \geq 0$ (i.e., both matrices are positive semi-definite), and $C_2 > 0$, i.e., C_2 is positive definite. Moreover, as before, the matrix pencil $\lambda C + G$ is assumed to be regular. It is easy to see that under these assumptions, $H(s)$ is positive real and thus the system is passive [10].

Theorem 2. Consider a passive system of the form (15). The ESVDMOR reduced system (13) is passive iff the inner reduction (12) is passivity preserving.

Proof. If we can show that $\hat{H}_r(s)$ in (13) is positive real, we have shown that the reduced system is passive, see Definition 2. The moments of (15) are

$$\mathbf{m}_i(s_0) = B^T (-(s_0 C + G)^{-1} C)^i (s_0 C + G)^{-1} B, \quad (16)$$

with $0 \leq s_0 \in \mathbb{R}$ and $\det(s_0 C + G) \neq 0$.

Following a technique which can be found, e.g., in [9] we define $J = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$. The

properties of G_1 , C_1 , and C_2 as well as the fact that $J = J^T$ and $J^2 = I$ lead to the following rules:

R1: $J = J^{-1}$,

R2: $JC = CJ$, hence $JCJ = C$,

R3: $(s_0C + G)^T = s_0C + JGJ = s_0JCJ + JGJ$,

hence $(s_0C + G)^{-T} = (s_0JCJ + JGJ)^{-1} = J^{-1}(s_0C + G)^{-1}J^{-1} = J(s_0C + G)^{-1}J$,

R4: $B = JB$, and

R5: for every matrix X and Y , $(-X^{-1}Y)^i = X^{-1}(Y(-X)^{-1})^iX$ holds.

A straightforward calculation employing these rules shows that

$$\begin{aligned}
\mathbf{m}_i^T(s_0) &= (B^T(-(s_0C + G)^{-1}C)^i(s_0C + G)^{-1}B)^T \\
&= B^T(s_0C + G)^{-T}\{(-(s_0C + G)^{-1}C)^i\}^T B \\
&\stackrel{(R5)}{=} B^T(s_0C + G)^{-T}\{(s_0C + G)^{-1}(C(-(s_0C + G)^{-1}))^i(s_0C + G)\}^T B \\
&= B^T(s_0C + G)^{-T}(s_0C + G)^T\{(C(-(s_0C + G)^{-1}))^i\}^T(s_0C + G)^{-T} B \\
&= B^T\{(C(-(s_0C + G)^{-1}))^i\}^T(s_0C + G)^{-T} B \\
&\stackrel{(C=C^T)}{=} B^T((-(s_0C + G))^{-T}C)^i(s_0C + G)^{-T} B \\
&\stackrel{(R3)}{=} B^T(-J(s_0C + G)^{-1}JC)^i(J(s_0C + G)^{-1}JB) \\
&\stackrel{(R5)}{=} B^TJ(s_0C + G)^{-1}(JC(-J(s_0C + G)^{-1}))^i(s_0C + G)J(J(s_0C + G)^{-1}JB) \\
&\stackrel{(R1,R2)}{=} B^TJ(s_0C + G)^{-1}(C(-(s_0C + G)^{-1}))^i(s_0C + G)(s_0C + G)^{-1}JB \\
&= B^TJ(s_0C + G)^{-1}(C(-(s_0C + G)^{-1}))^iJB \\
&\stackrel{(R4)}{=} B^T(s_0C + G)^{-1}(C(-(s_0C + G)^{-1}))^iB \\
&\stackrel{(R5)}{=} B^T(-(s_0C + G)^{-1}C)^i(s_0C + G)^{-1}B \\
&= \mathbf{m}_i(s_0).
\end{aligned}$$

It follows from (6) that $M_I = M_O$, such that

$$V_{I_{r_i}}^T = V_{O_{r_o}}^T = V_r^T \quad (17)$$

with the help of (7). It is easy to see that

$$\hat{H}(s) = V_r B_r^T (G + sC)^{-1} B_r V_r^T, \quad (18)$$

with B_r analogously to (9). Hence, if the model reduction method used in (12) leads to a positive real transfer function of the reduced-order model, passivity is preserved.

□

Remark 1. As a simple numerical example we show a RC circuit called *rc549*, which is also investigated in [22] and Chapter ?? of this volume. It is provided by the Infinoen Technologies AG, Munich, Germany. The order of the corresponding descriptor system is $n = 141$, the number of terminals is $m = 70$. For sim-

Fig. 1 Smallest eigenvalues of the Hermitian part of the transfer function of *rc549* depending on the frequency ω .

plicity, we use $H_{DC} = m_0(s = 0)$ as basis for the matrices in (6) and we abstain from the reduction step in (12). Following Definition 2, we show that the Hermitian part of the transfer function on the imaginary axis is positive semi-definite, i. e., $H_H = H(j\omega) + H(j\omega)^* \geq 0$. Figure 1 shows the smallest eigenvalue of H_H of the original transfer function and of the terminal reduced transfer functions $H_r(s)$. Although the frequency range is much larger than important in applications, this smallest eigenvalue grows depending on how much terminals are reduced. The relative difference of these smallest eigenvalues of the reduced systems to those of the original system is shown in Figure 2. This difference depends on the spectrum of $H(s)$, but does not necessarily play a role in the aspect of passivity preserving. Note, that the transfer function in (13) has additional zero eigenvalues which do not destroy the positive semi-definiteness of the transfer function.

3.3 Reciprocity

Another important property of MOR methods is reciprocity preserving, which is sufficient for synthesis of the reduced-order model as circuit. We assume the same setting as in Section 3.2. An appropriate definition can be found, e. g., in [20].

Fig. 2 Relative difference correlated to the largest eigenvalue of the original transfer function.

Definition 3. A transfer function (14) is *reciprocal* if there exist $m_1, m_2 \in \mathbb{N}$ with $m_1 + m_2 = m$, such that for $\Sigma_e = \text{diag}(I_{m_1}, -I_{m_2})$ and all $s \in \mathbb{C}$ not being poles of $H(s)$, it holds

$$H(s)\Sigma_e = \Sigma_e H^T(s).$$

The matrix Σ_e is called *external signature* of the system. A descriptor system is called reciprocal if its transfer function is reciprocal.

As a consequence, a transfer function of a reciprocal system has the form

$$H(s) = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ -H_{12}^T(s) & H_{22}(s) \end{bmatrix}, \quad (19)$$

where $H_{11}(s) = H_{11}^T(s) \in \mathbb{R}^{m_1, m_1}$ and $H_{22}(s) = H_{22}^T(s) \in \mathbb{R}^{m_2, m_2}$, which means that the transfer function possesses a non-Euclidian symmetry.

Theorem 3. Consider a reciprocal system of the form (15). The ESVD MOR reduced system (13) is reciprocal iff the inner reduction (12) is reciprocity preserving.

Proof. Due to the reciprocity of the original system, the corresponding transfer function (14) has the structure given in (19). Equation (18) shows that none of the steps in ESVD MOR destroys this symmetric structure if (12) preserves reciprocity.

□

4 Remarks and Outlook

We have reviewed the ideas of ESVD MOR and shown that this approach is stability, passivity, and reciprocity preserving under reasonable assumptions described in Section 3.2. As a direction of future research, it would be desirable to have a proof of passivity preservation for more general settings, e.g., a more general structure than in (15).

In [5] it is pointed out that an explicit computation of the moments in (3) would be numerically unstable and too expensive. In order to avoid this problem, a numerical solution by using the truncated SVD based on the implicitly restarted Arnoldi method is presented there. As an alternative, we also investigate the Jacobi-Davidson approach (JDSVD) [12] for computing the TSVD. In this context, the computation of the residual and the solution of the correction equation within JDSVD are based on iterative methods. This offers a large potential for increased efficiency in ESVD MOR due to the usual structures of circuit equations that can be exploited here.

An automatic error control which regulates the number of significant singular values as well as the best choice of the decomposition method and the number of used moments are other interesting topics for future research.

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