Otto-von-Guericke-Universität Magdeburg
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Einführung Numerische Linear Algebra – Biconjugated Gradient Method.

Algorithmus: Biconjugate Gradient Method (BICG)

Wähle x_0, $r_0 = b - Ax_0$, wähle initiales Residuum für adjungiertes LS, z.B., $s_0 = c - A^T y0$.
$p_0 = r_0$, $q_0 = s_0$.
for $k = 0, 1, \ldots$ **do**
  $\alpha_k = \frac{\langle s_k, r_k \rangle}{\langle q_k, Ap_k \rangle}$
  $x_{k+1} = x_k + \alpha_k p_k$
  $y_{k+1} = y_k + \alpha_k q_k$ (approx. Lösung des adjungierten LS $A^T y = c$, wenn benötigt)
  $r_{k+1} = r_k - \alpha_k A p_k$
  $s_{k+1} = s_k - \alpha_k A^T q_k$
  $\beta_{k+1} = \frac{\langle s_{k+1}, r_{k+1} \rangle}{\langle s_k, r_k \rangle}$
  $p_{k+1} = r_{k+1} + \beta_{k+1} p_k$
  $q_{k+1} = s_{k+1} + \beta_{k+1} q_k$
end for

Eigenschaften

- $\text{span}\{r_0, r_1, \ldots, r_k\} = \text{span}\{p_0, \ldots, p_k\} = \mathcal{K}_{k+1}(A, r_0)$
- $\text{span}\{s_0, s_1, \ldots, s_k\} = \text{span}\{q_0, \ldots, q_k\} = \mathcal{K}_{k+1}(A^T, s_0)$
- Petrov-Galerkin-Bedingungen: $r_k \perp \mathcal{K}_k(A^T, s_0)$, $s_k \perp \mathcal{K}_k(A, r_0)$
- Biorthogonalität: $\langle r_k, s_j \rangle = \langle Ap_k, q_j \rangle = 0$ for $j \neq k$.
- Breakdowns, z.B., wenn $\langle q_k, Ap_k \rangle = 0$ oder $\langle r_k, s_k \rangle = 0$.